TItle: Godunov type numerical method for Conservation laws with a flux function discontinuous in space.

Abstract. We study scalar conservation law:

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(k(x), u) = 0 \text{ for } x \in I\!R, t > 0 \\ u(x,0) = u_0(x), x \in I\!R \end{cases} \quad (1)$$

where the function $k(x)$ is discontinuous. This type of problem appears for example in modelling of two phase flow in porous media, in sedimentation problem and in traffic flow. Corresponding Hamilton-Jacobi equation appears in Shape from Shading. Because of the discontinuity of the flux in space, the Kruzkov method does not guarantee existence of a weak solution and even if the solution exists, it may not be unique. Here we propose a Godunov type numerical method for which we prove the convergence. Also it is shown that limit of this approximate solution is a weak solution of (1). At the interface we introduce interface entropy condition which pick up the correct solution and very simple formula is given for the interface numerical flux. Numerical results are presented for problems arising in two phase flow and for Hamilton-Jacobi equation which arise in Shape from Shading.