SINGULAR LIMITS FOR HYPERBOLIC CONSERVATION LAWS : GENERAL DIFFUSION, RELAXATION AND BOUNDARY LAYERS

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Abstract

Hyperbolic systems of conservatin laws in one space dimension appear in many Physical applications when the diffusion effects are neglected. They takes the form

$$u_t + F(u)_x = 0$$

where the flux F is a smooth function from a convex subset of R^n to R^n. There is a well developed theory for the pure initial value problem for this system when it is strictly hyperbolic. There are many unresolved questions when the diffusion effects are also taken into account. We consider

$$u_t + F(u)_x = \epsilon(B(u)u_x)_x$$

and variants of it for the pure initial value problem and boundary value problems in the limit ϵ tending to 0 and review some recent developments in this topics. This include recent joint work with Philippe LeFloch on self-similar approximations,relaxation appxoximations, boundary layers and formulation of the boundary condition using boundary layers.