The Theory of exact relations for composites

G.Milton

Department of Mathematics, The University of Utah, Salt Lake City, Utah 84112, U.S.A.

This is joint work with Yury Grabovsky and Dan Sage. Typically, the elastic and electrical properties of composite materials are strongly microstructure dependent. So it comes as a nice surprise to come across exact formulae for (or linking) effective moduli that are universally valid no matter what the microstructure. Such exact formulae provide useful benchmarks for testing numerical and actual experimental data, and for evaluating the merit of various approximation schemes. Classic examples include, Hill's formulae for the effective bulk modulus of a two-phase mixture when the phases have equal shear moduli, Levin's formulae linking the effective thermal expansion coefficient and effective bulk modulus of two-phase mixtures, and Dykhne's result for the effective conductivity of an isotropic two-dimensional polycrystalline material. Here we present a systematic theory of exact relations embracing the known exact relations and establishing new ones. The search for exact relations is reduced to a search for tensor subspaces satisfying a certain algebraic condition. One of many new exact relations is for the effective shear modulus of a class of three-dimensional polycrystalline materials.