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Coupled simulation for atmospheric and water current models for lakes and ponds and related topics

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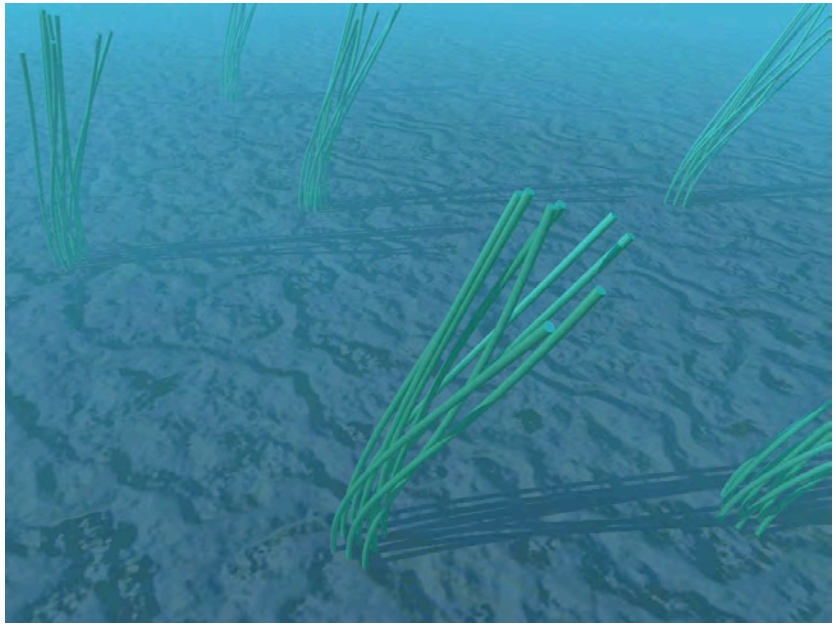

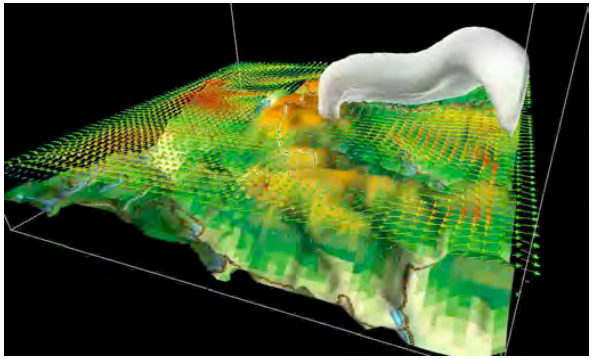

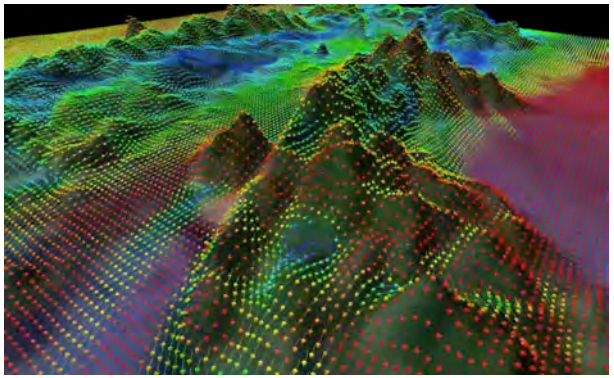

Contents

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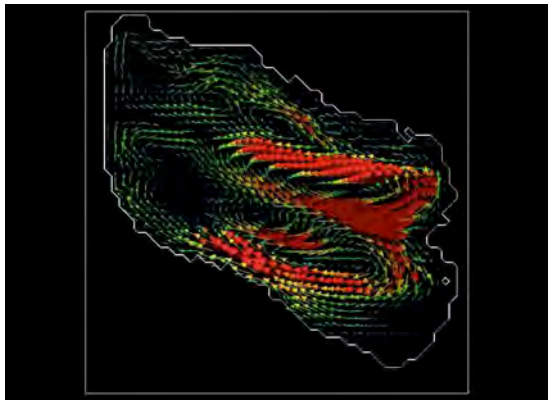
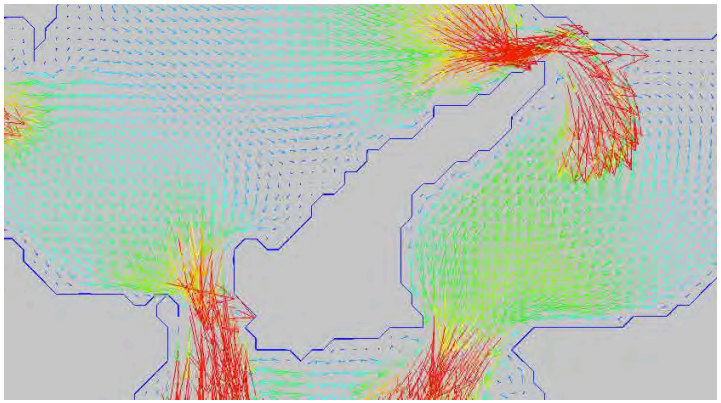
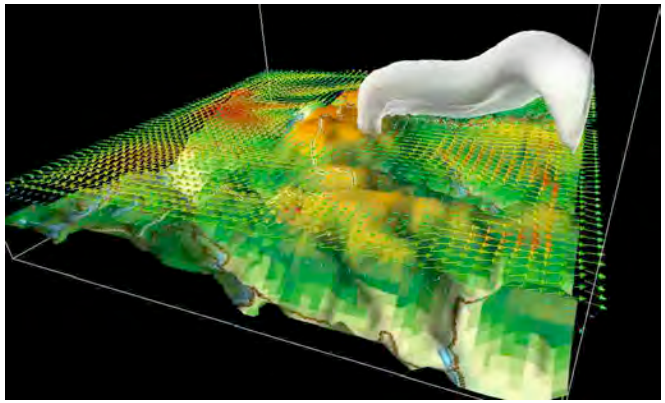
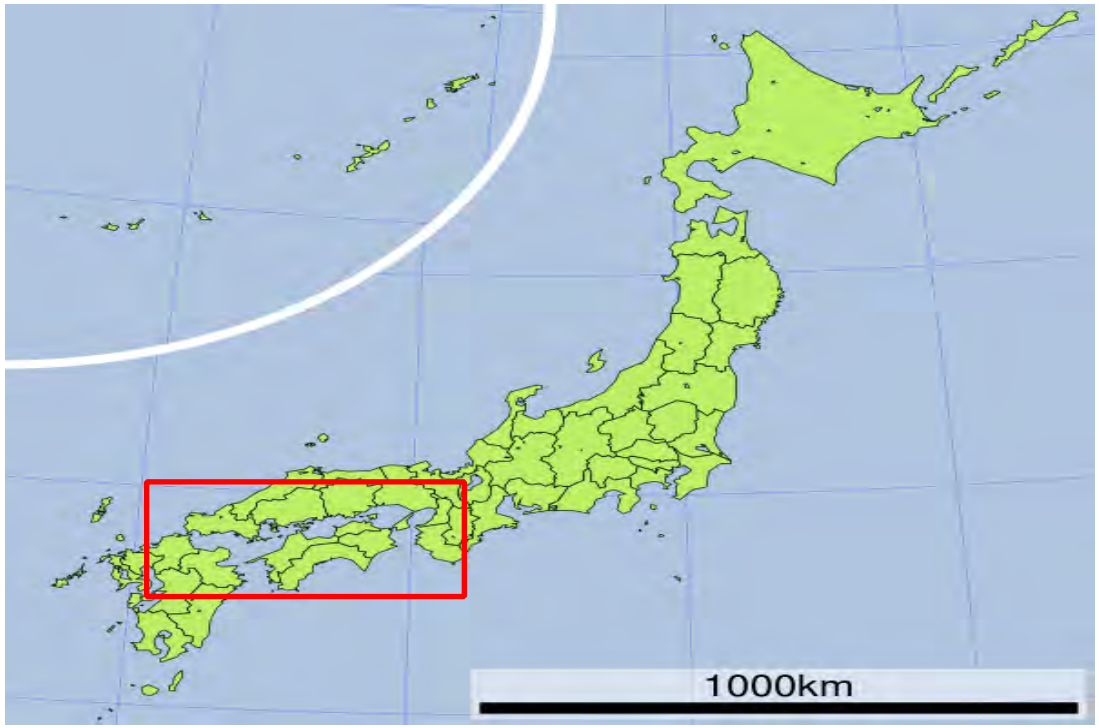
- Coupled simulation for Lake Pyykösjärvi
- Fluid-structure interaction between seagrass and water current
- Convection-diffusion of toxic materials in mountainous area





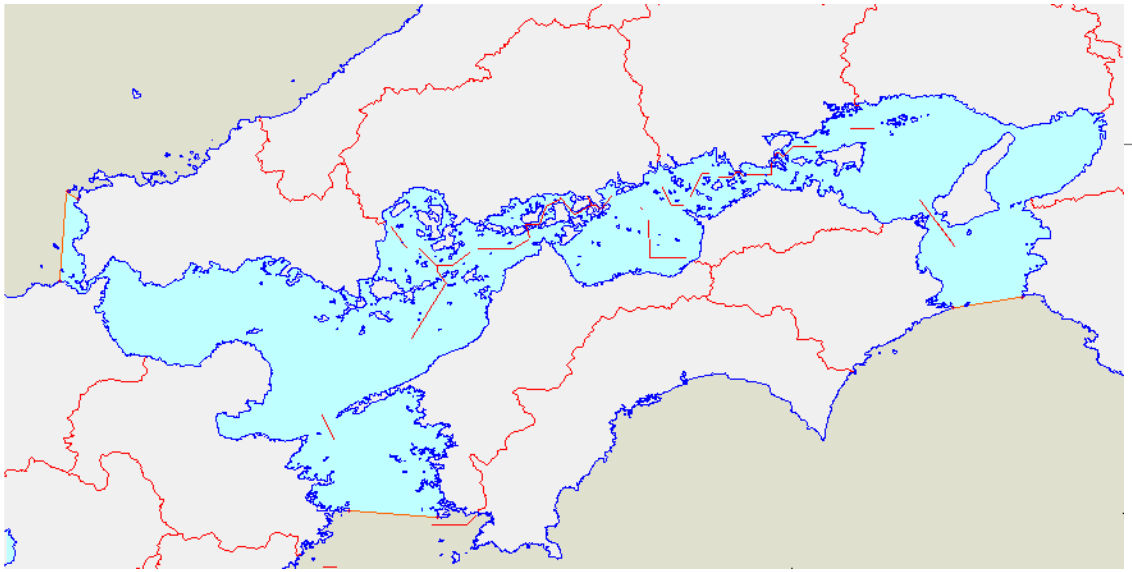


Tidal flow simulation for Seto Inland Sea



Scale
- 500 km (east to west)
- 20 – 60 km (south to north)

Complicated tidal flow and tidal height distribution

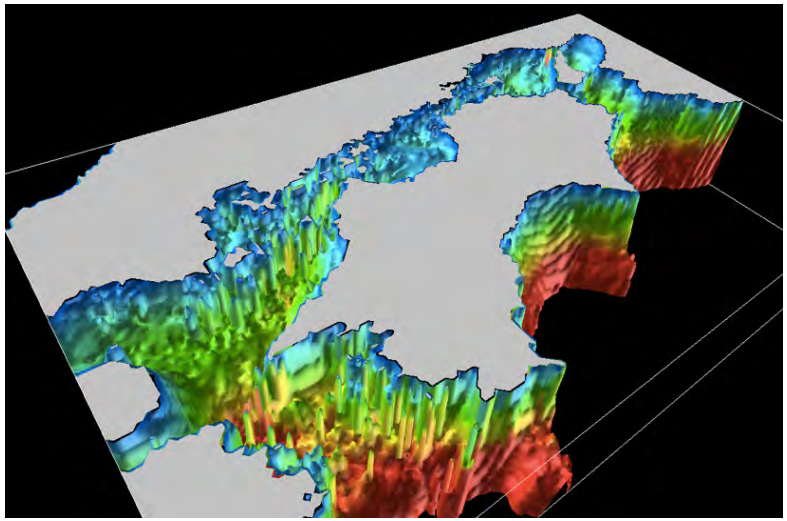


Sometimes, big earthquakes are followed by tsunamis. Tidal level and tidal flow distribution are important information.

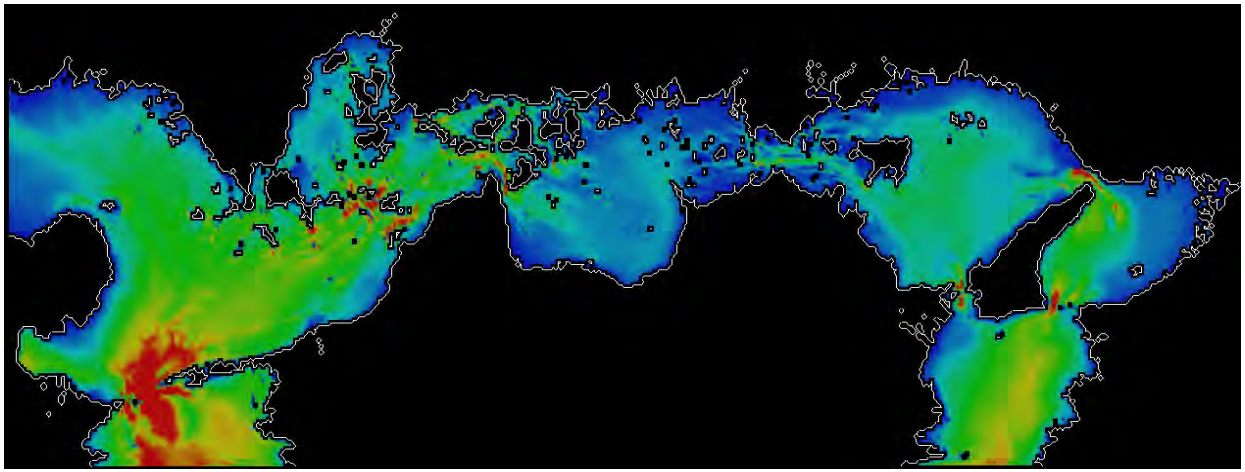
Though the scale of the inland sea is not so big, tidal motion is very complicated. Actually, it shows opposite phases in different location in the inland sea.

# of islands: 600
Average depth: 37.3 m
The deepest: 105 m.

Maximum tidal flow : 5m/s
Maximum difference between high and low tide levels : 2m



Bathymetry



Open boundary conditions

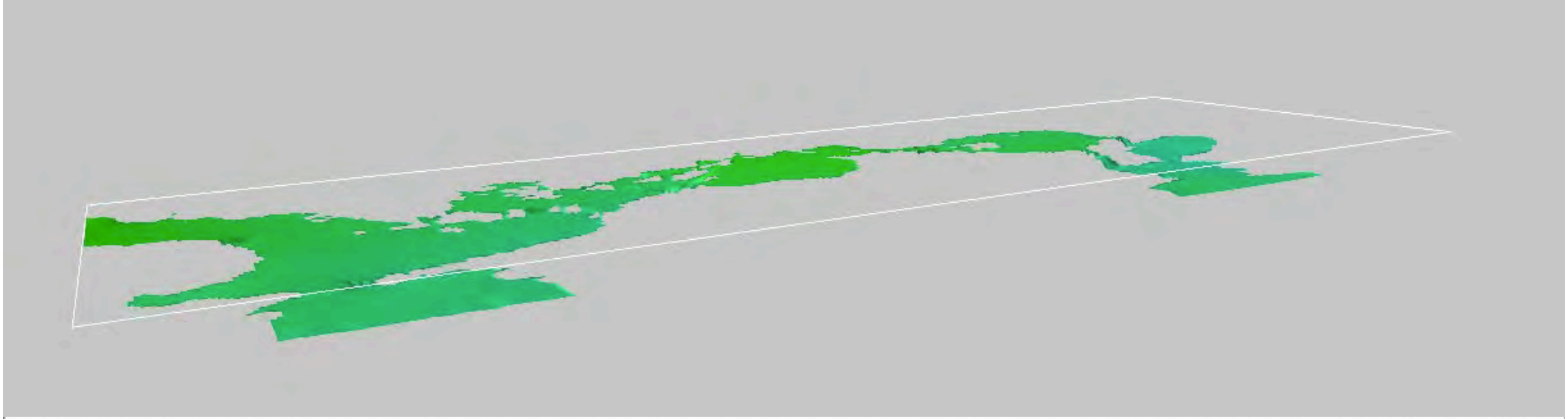
Boundary

Γ :

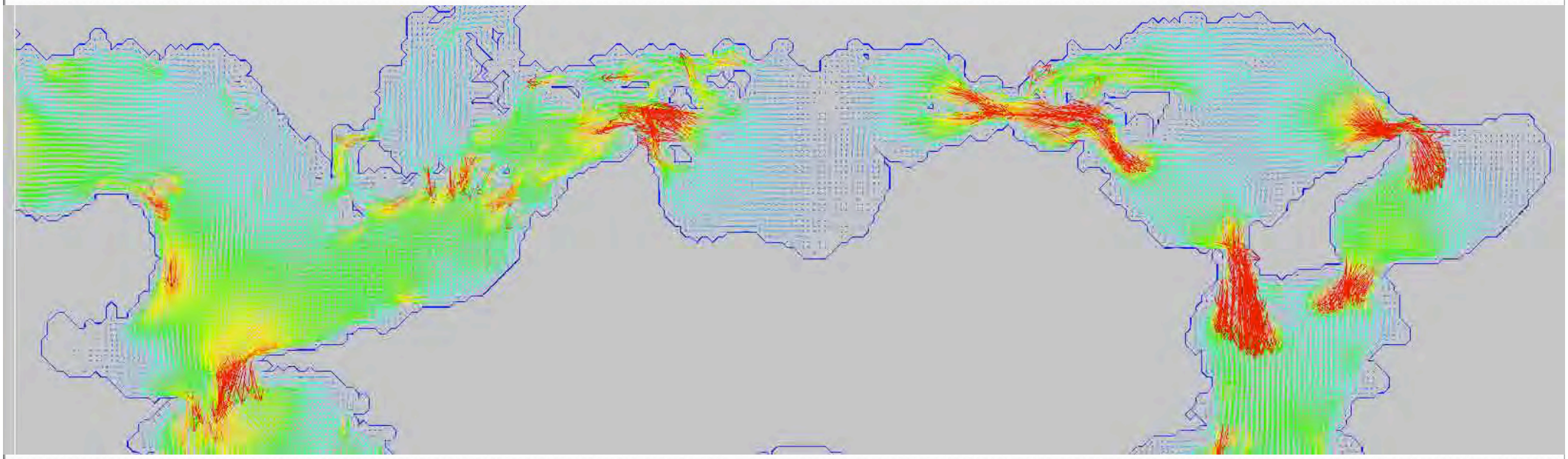
Computation area

Ω :

M_2, S_2, K_1, O_1 tidal components are used.

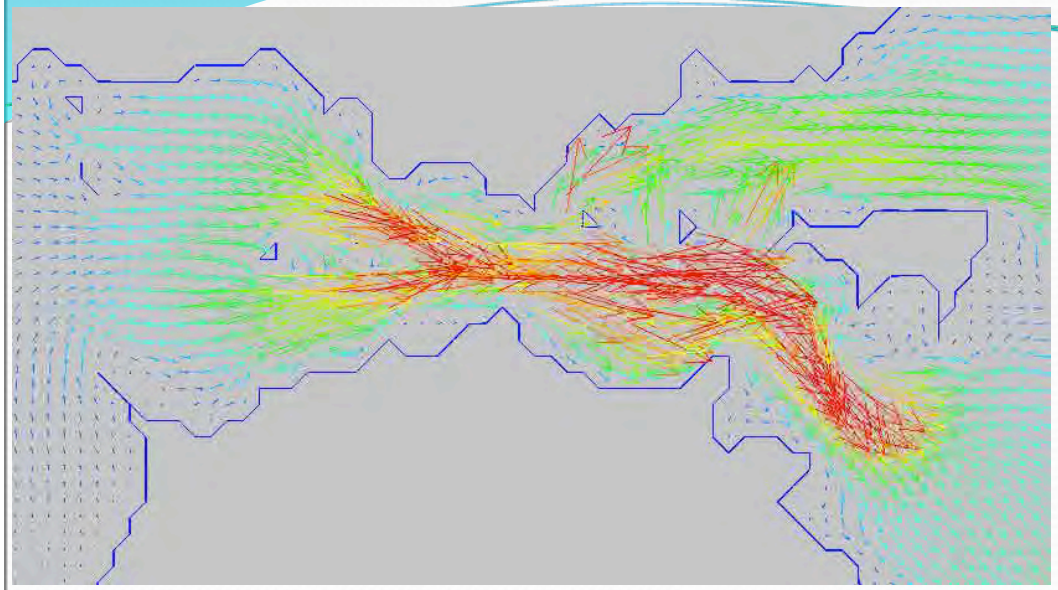


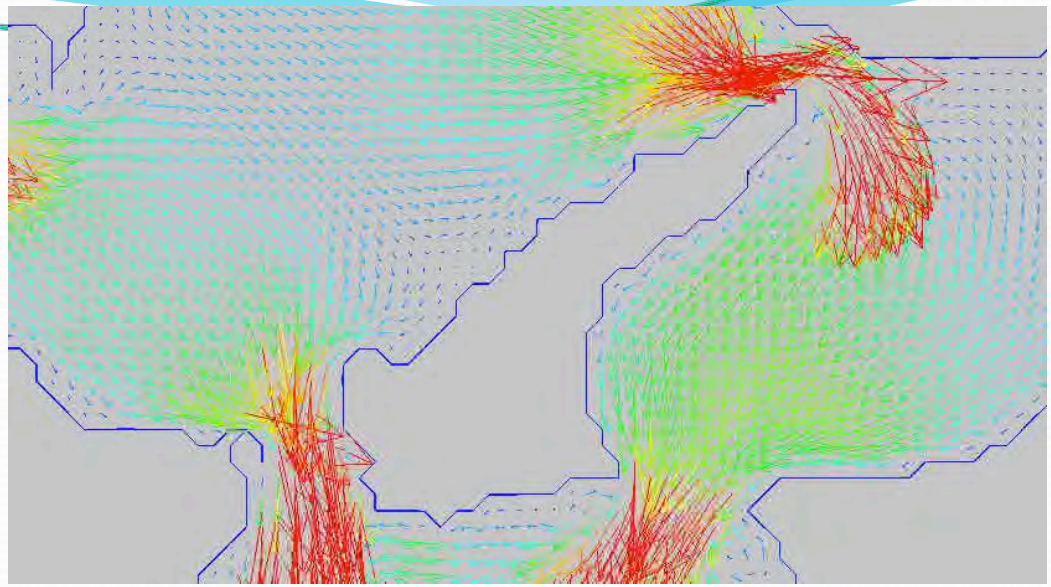
Tide level

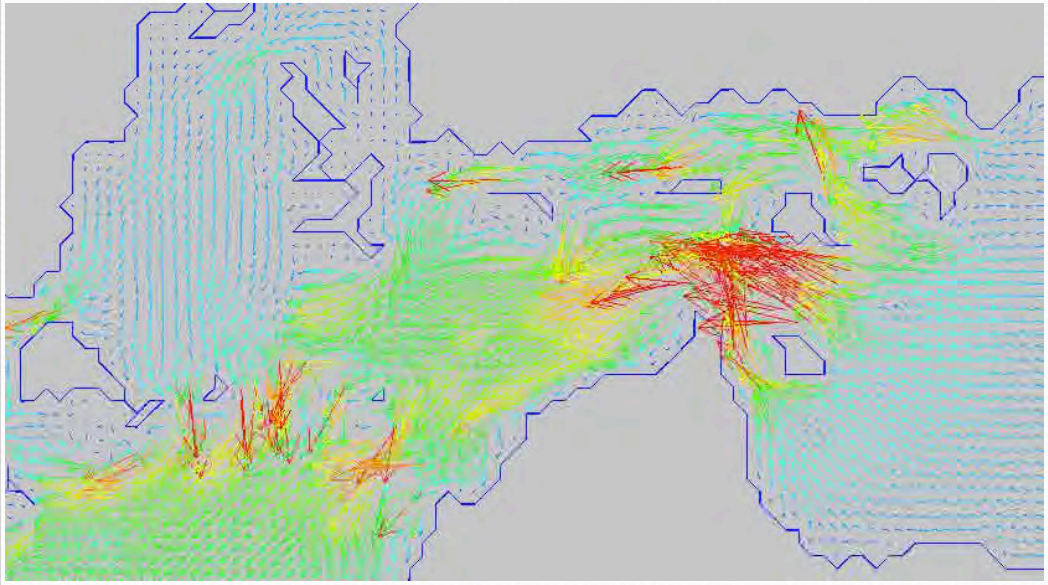


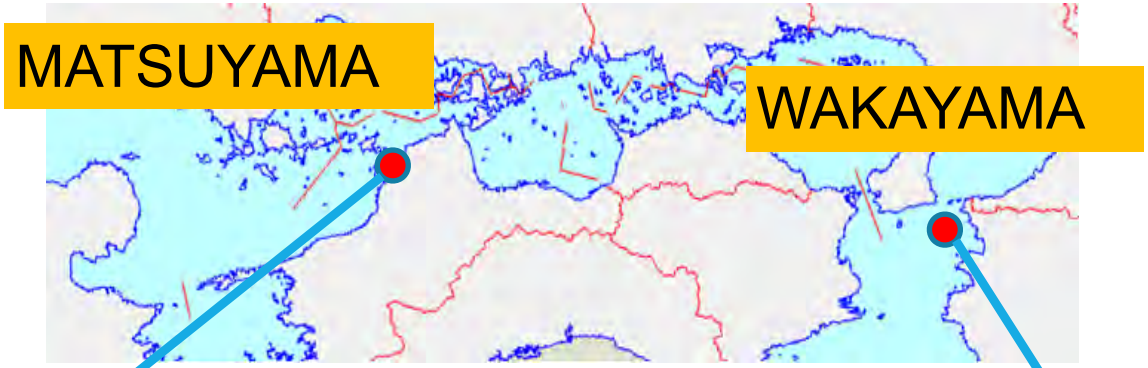
Tidal flow

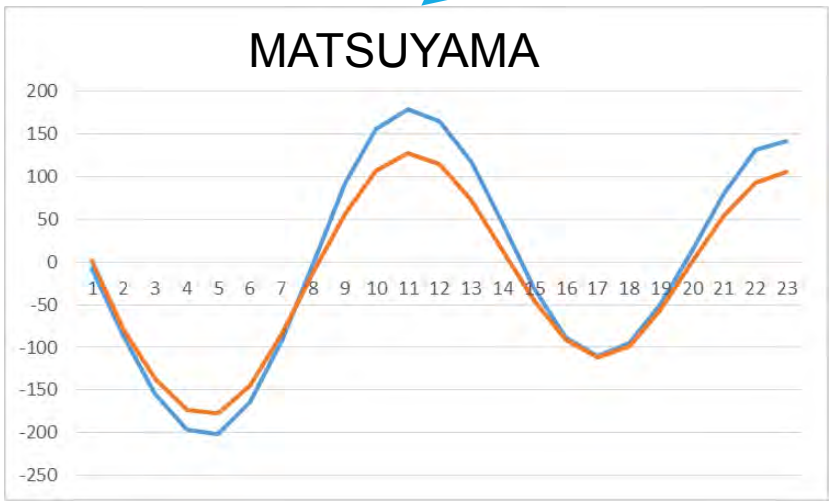
Low  High

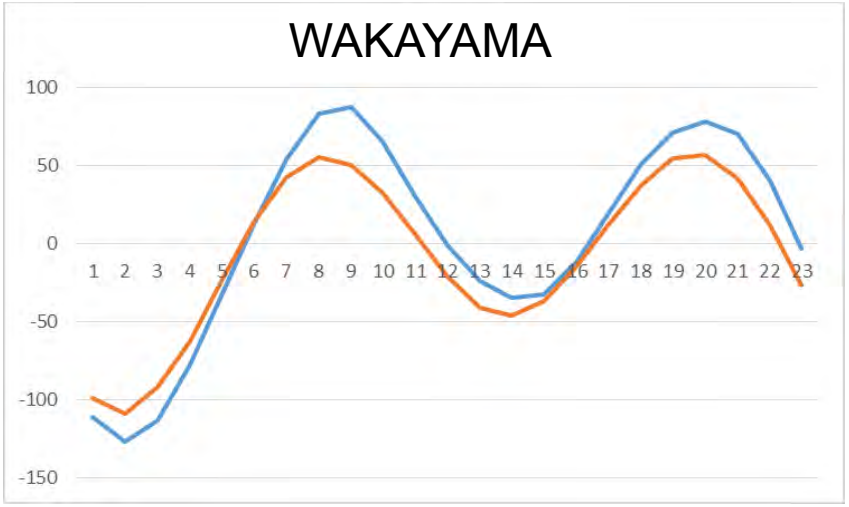










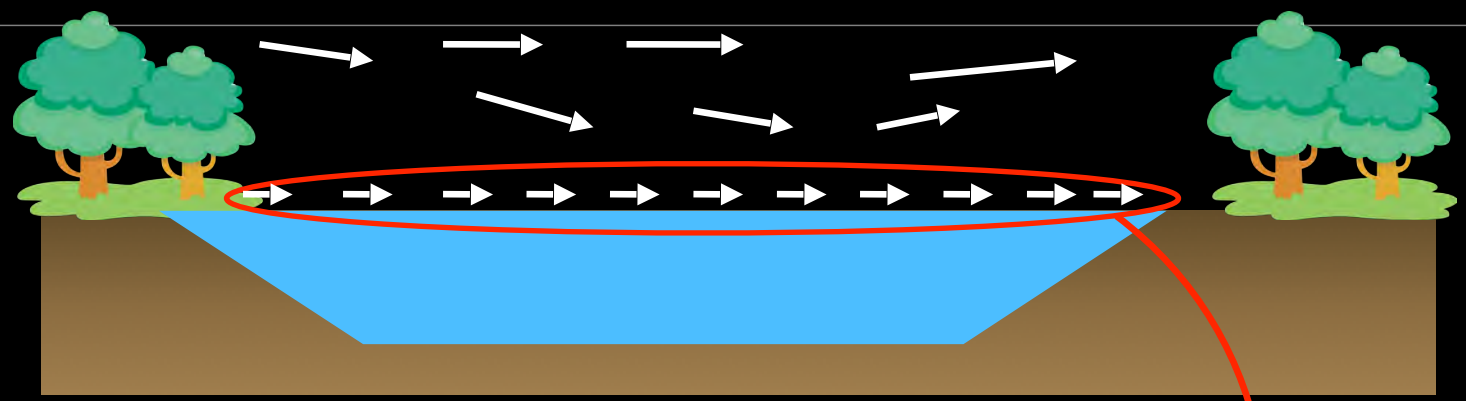


Observed value　　Calculation result

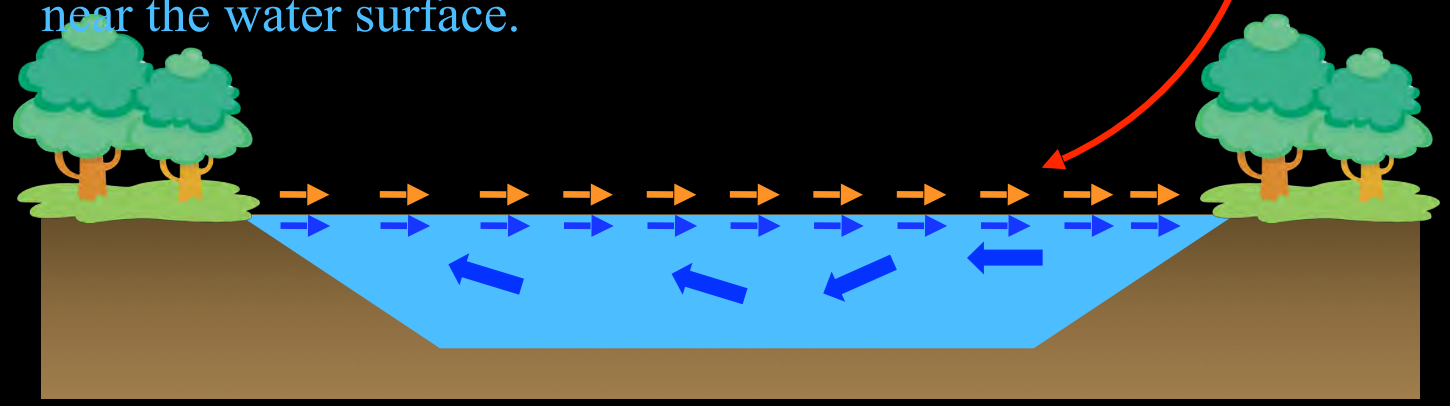
The comparison is in good agreement for tidal phase between computational results and observed data

Wind - water current coupling for Pyykösjärvi

Firstly the air flow is calculated.



Then boundary conditions for water flow are given by using the wind velocity near the water surface.



Governing equations for air flow

- Navier-Stokes equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -C_p \Theta \nabla \Pi' + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g}\frac{\theta'}{\Theta}$$

- Continuity equation

$$\nabla \cdot \boldsymbol{u} = 0$$

- Heat equation

$$\frac{\partial \theta'}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta' = \lambda \nabla^2 \theta' + \frac{L_v \cdot m}{\rho C_p}$$

- Transport equation of moisture

$$\frac{\partial q}{\partial t} + (\boldsymbol{u} \cdot \nabla) q = -m$$

$\boldsymbol{u}$: wind velocity $[m/s]$

C_p : specific heat capacity at a constant pressure $[J/kg \cdot K]$

Θ : horizontally – averaged potential temperature $[K]$

Π : Exner's function [-]

ν : kinematic viscosity $[m^2/s]$

$\boldsymbol{g}$: gravity $[m/s^2]$

θ' : reduced potential temperature $[K]$

T : temperature $[K]$

λ : thermal diffusivity $[m^2/s]$

L_v : latenth heat of vaporization $[J/kg]$

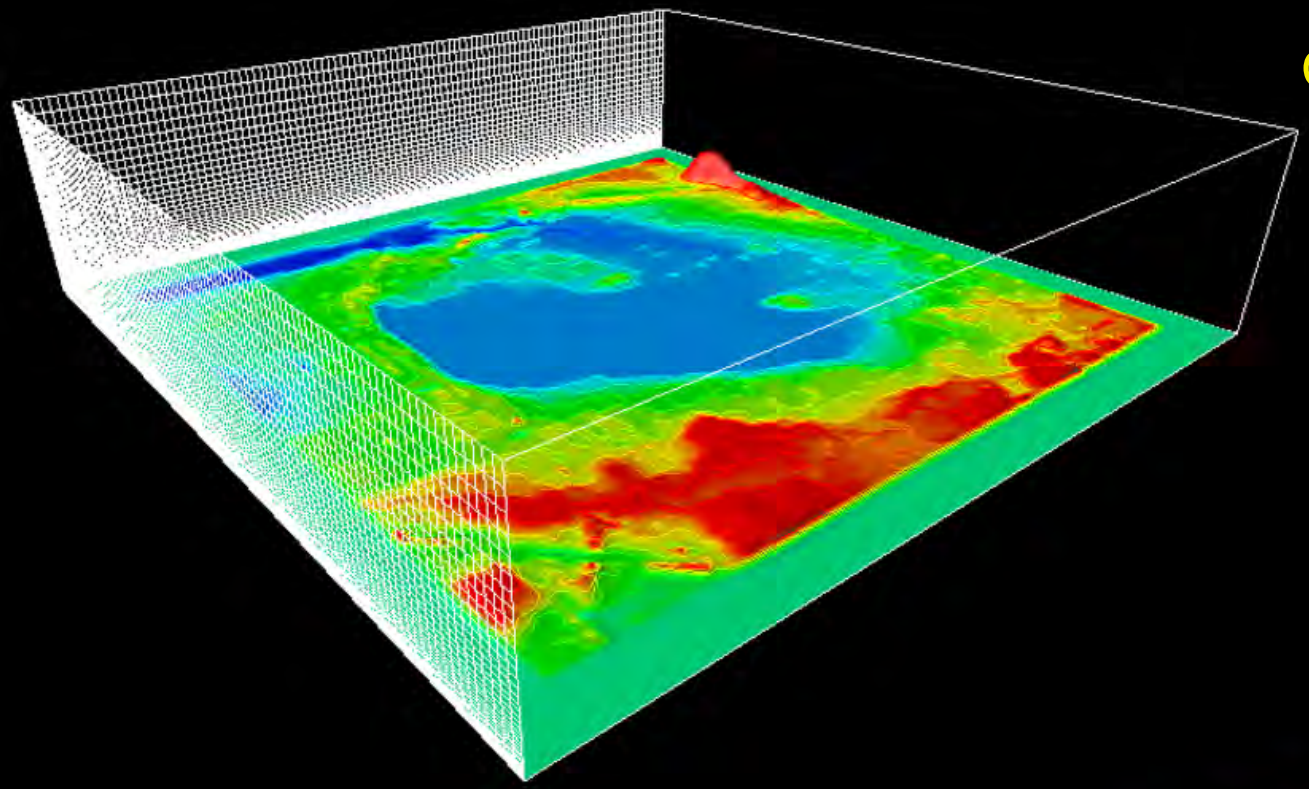
m : condensation $[kg/m^3 \cdot s]$

ρ : density of air $[kg/m^3]$

q : water vapor content $[kg/m^3]$

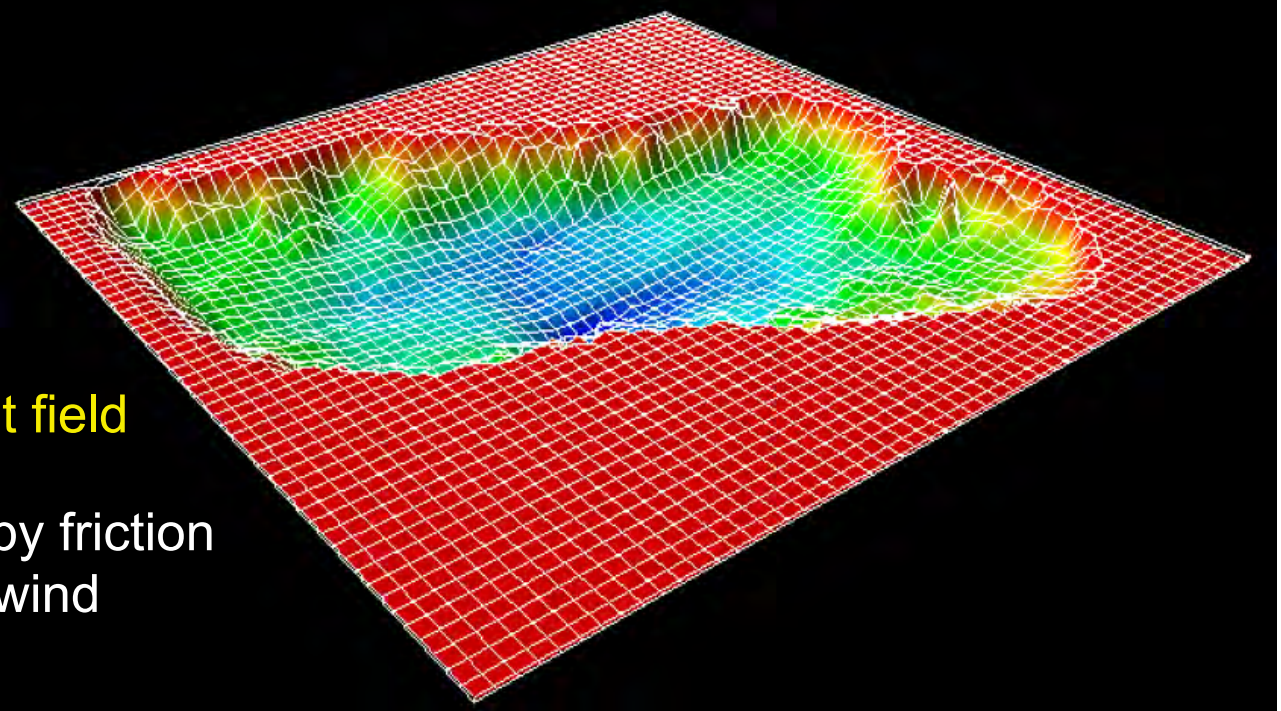
σ-coordinate system is employed using the following equation.

$$\frac{\partial f}{\partial x_k} = \frac{\partial f}{\partial \xi_i}\frac{\partial \xi_i}{\partial x_k}$$



Computation of wind field

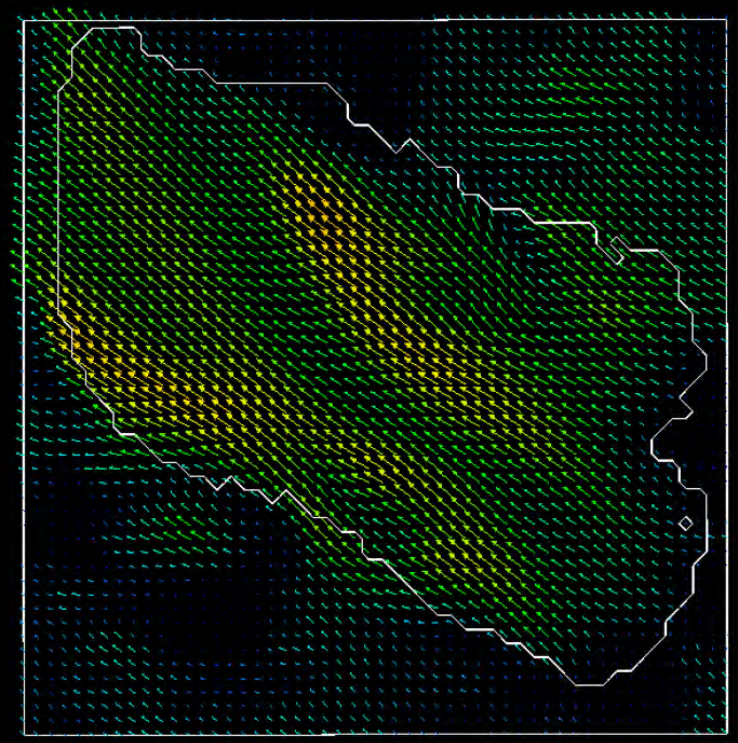
Forest around the lake is considered to be a porous media



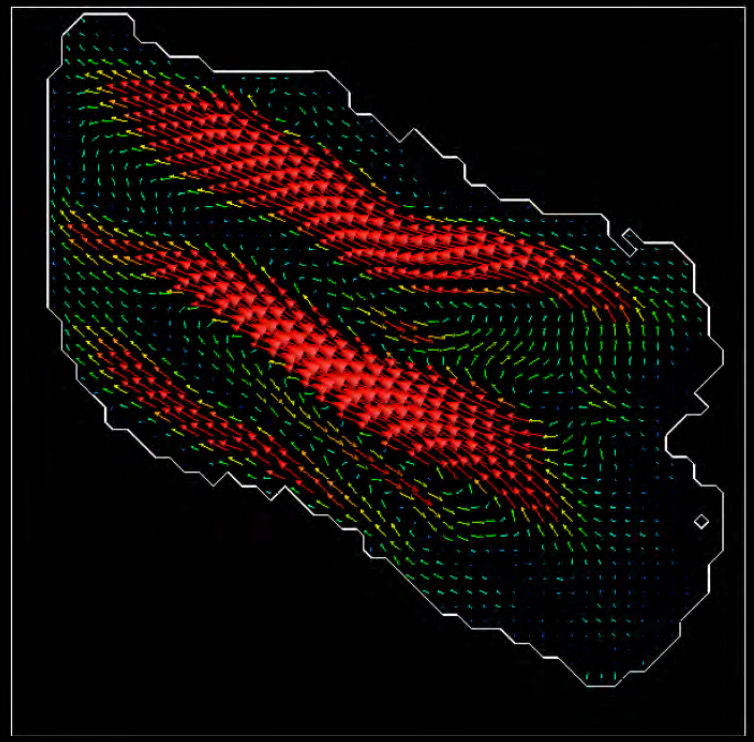
Computation of water current field

Flow is driven by friction force from the wind

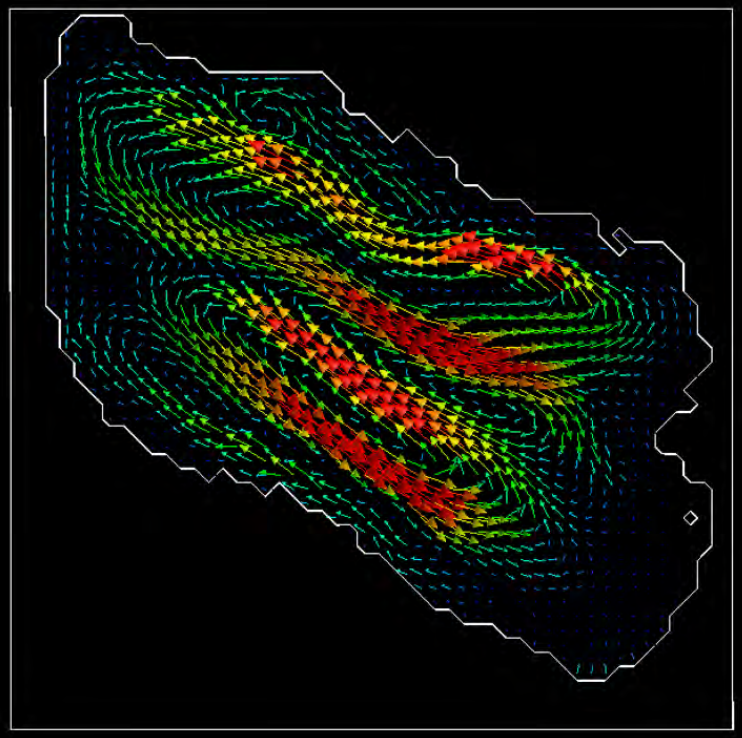
Coupling with atmospheric model, 1 pm, 21/09/2011



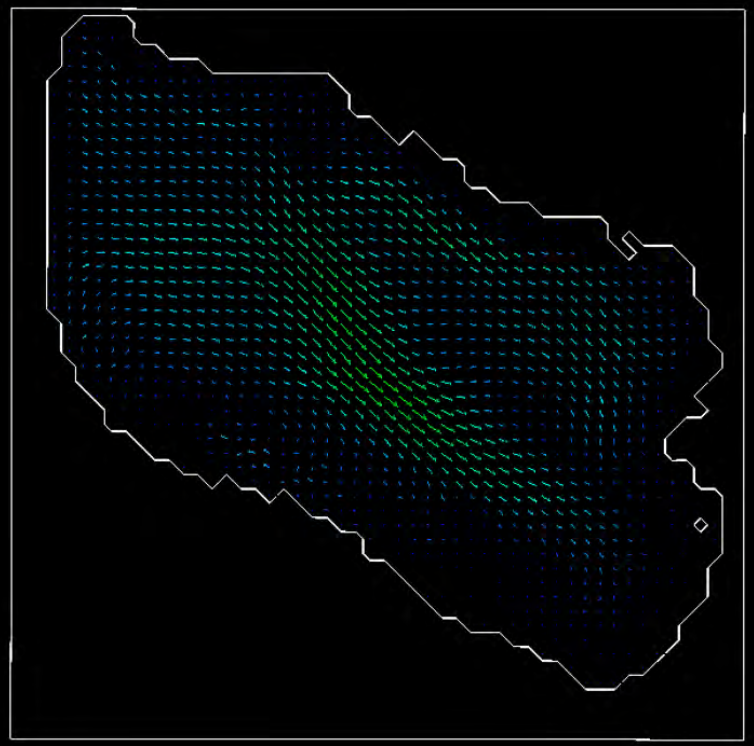
Wind



Water (surface)

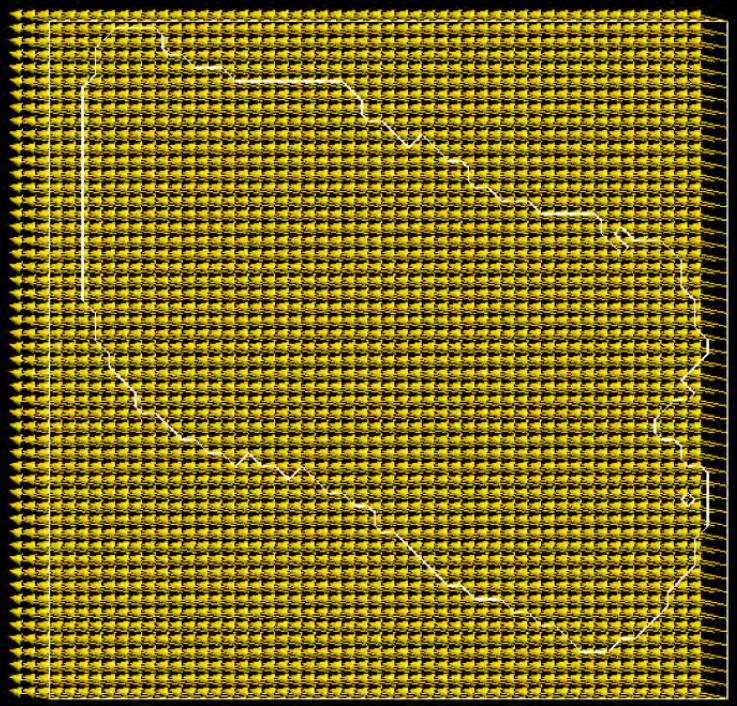


Water (medium)

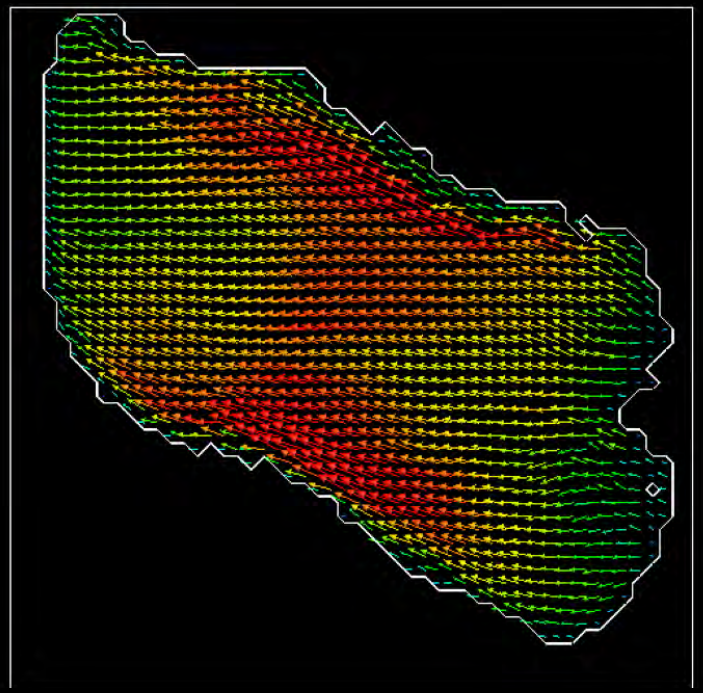


Water (bottom)

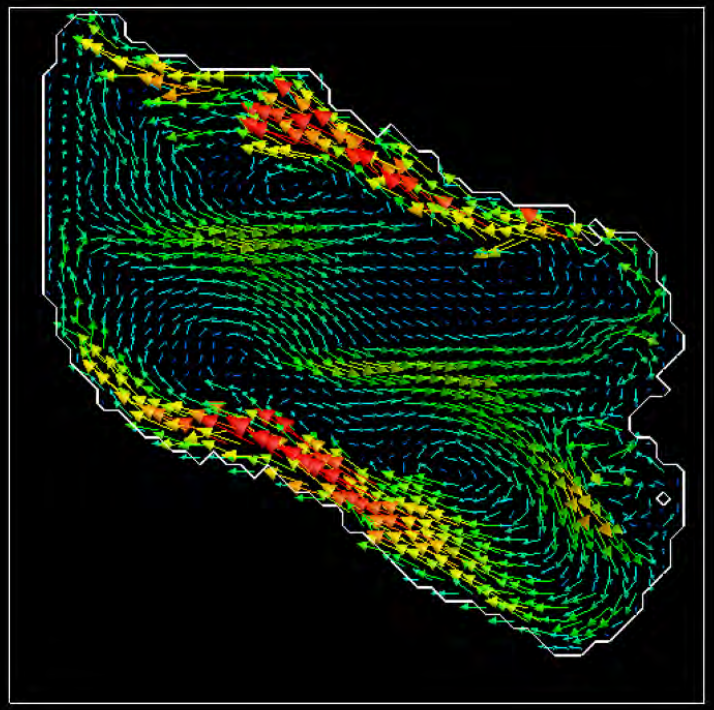
Homogeneous wind, 1 pm, 21/09/2011



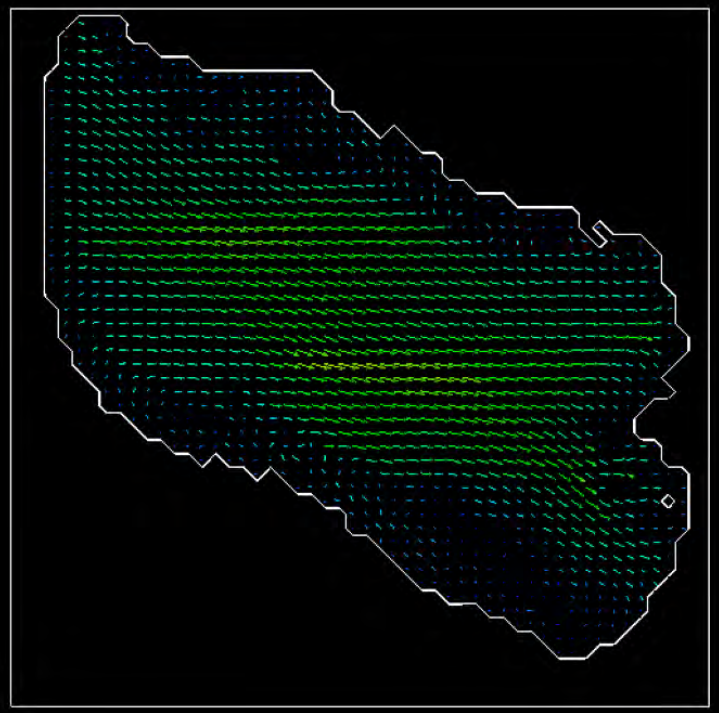
Wind



Water (surface)



Water (medium)

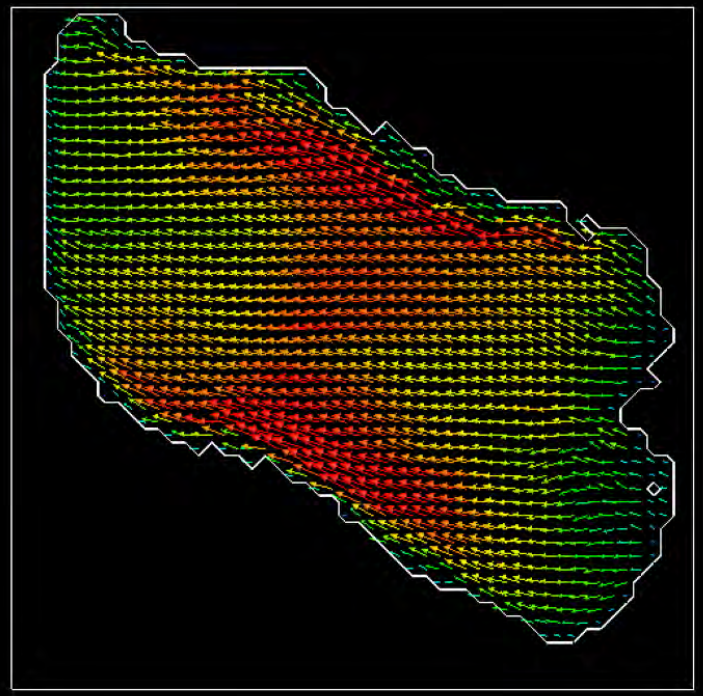


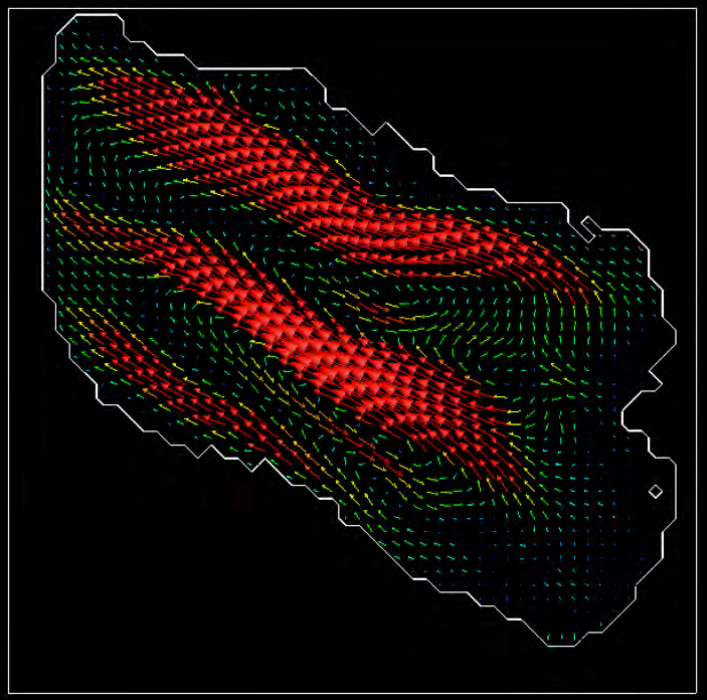
Water (bottom)

Homogeneous wind

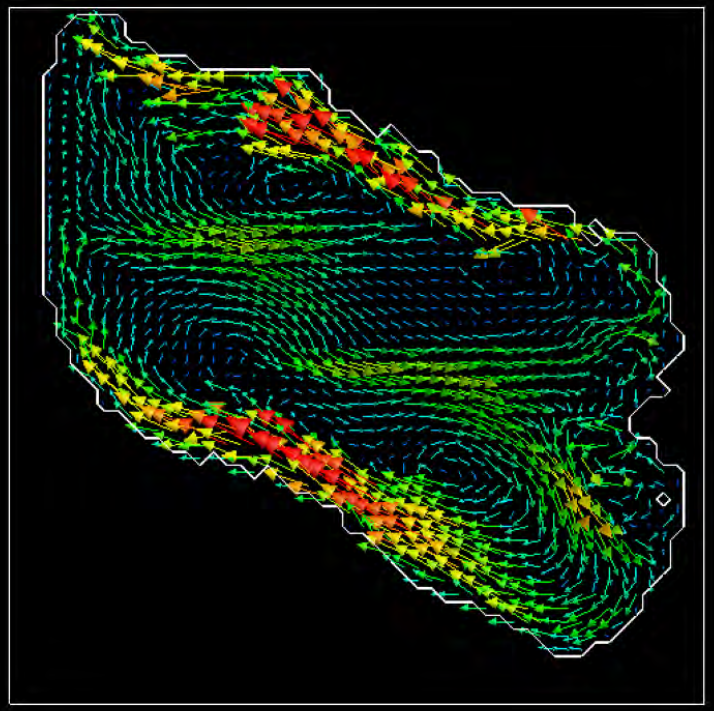
Coupling with atmospheric model

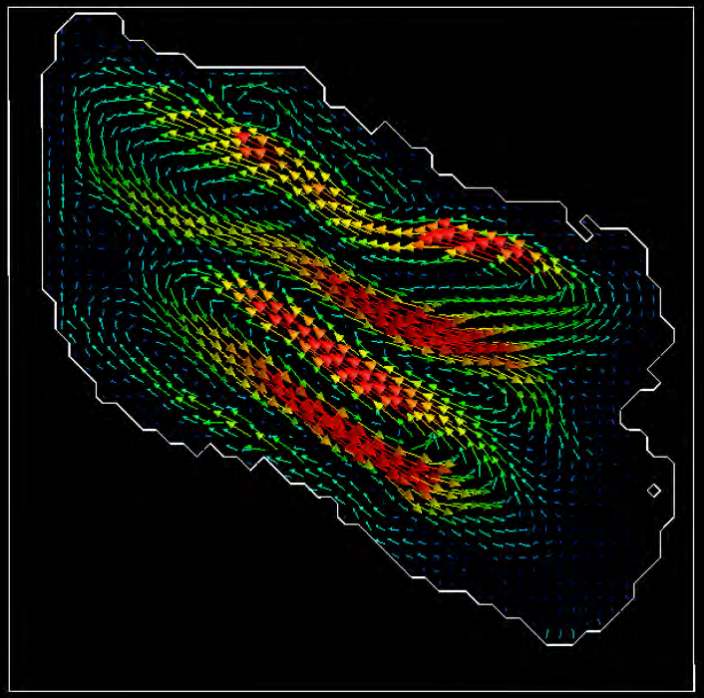
Water (surface)





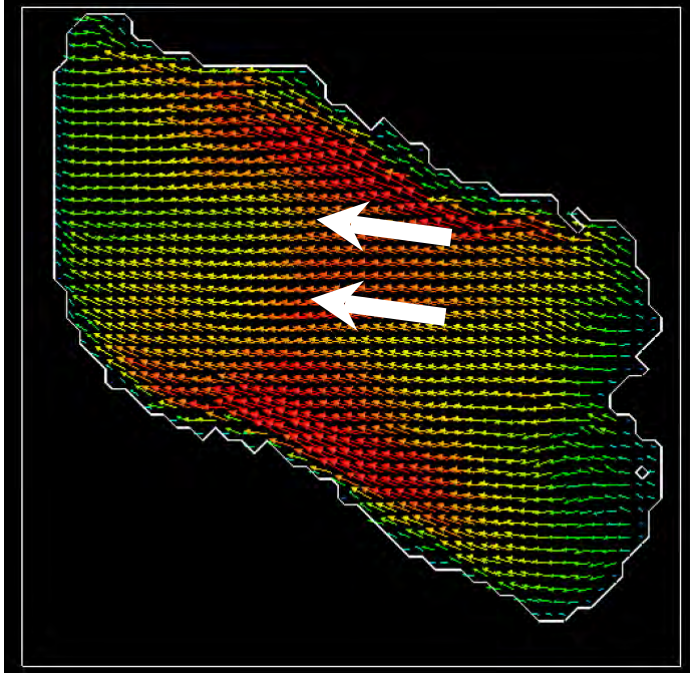
Water (medium)



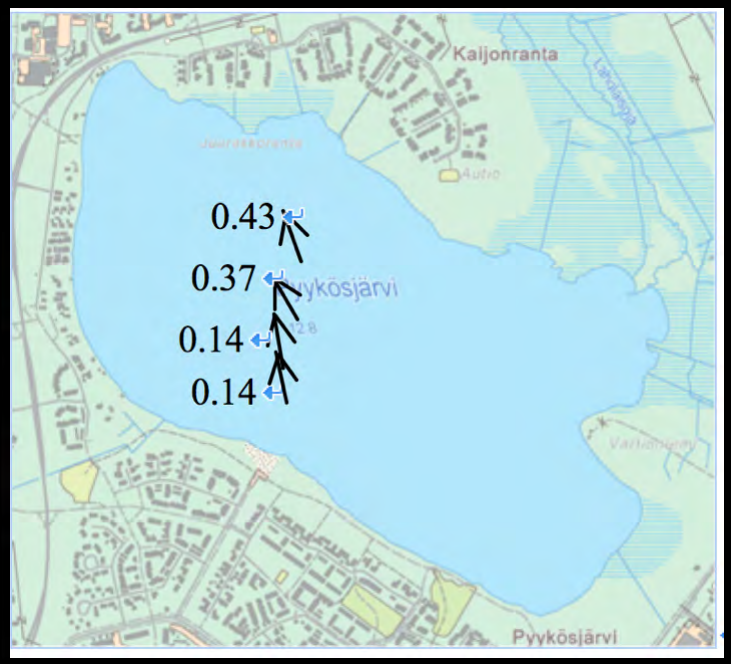


Comparing with water current measurements by using drifter

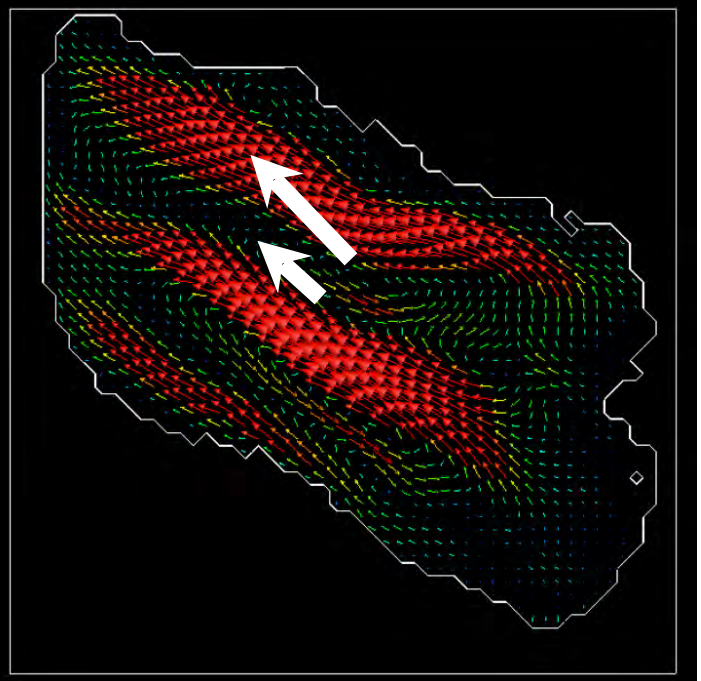
Homogeneous wind without coupling



Drifter experiment by N. Liukko (SYKE)



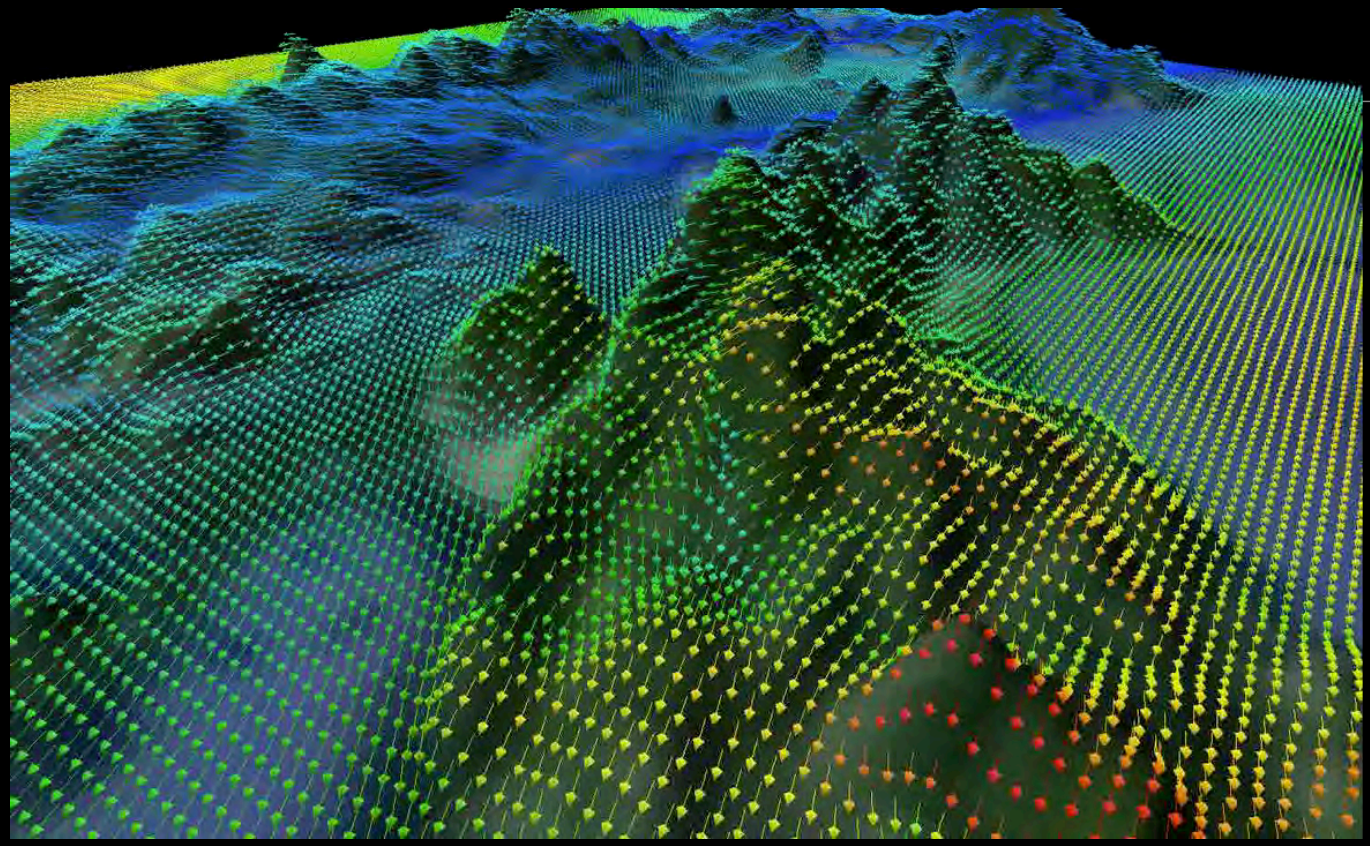
Coupling with atmospheric model



- Water current is affected by wind distribution on the lake.
- Wind distribution is also affected by forests etc around the lake.

Three generations of the nesting in the Seto Inland Sea case

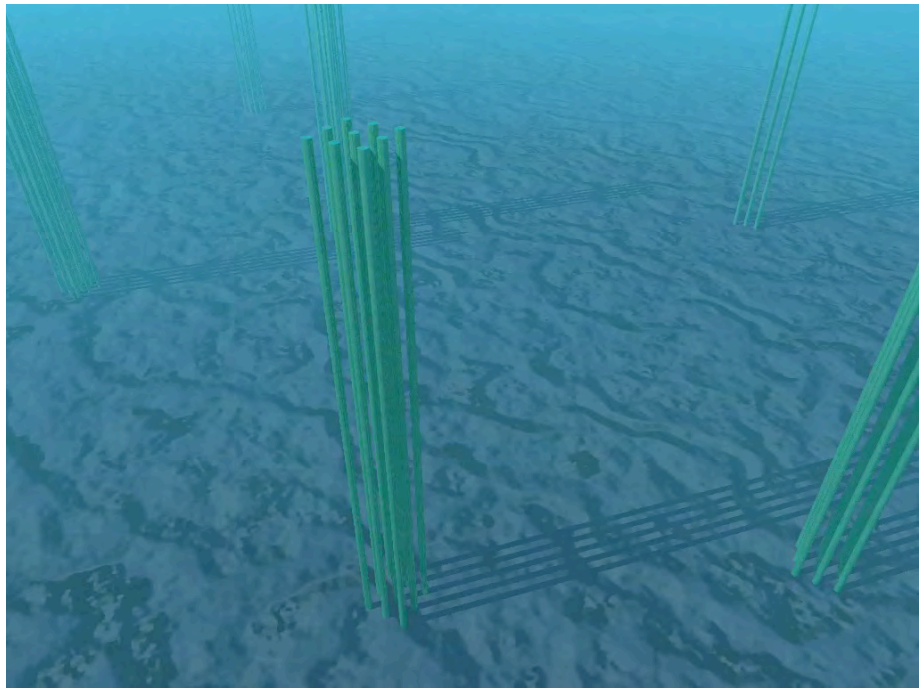
Computation by Shota Doi



Grandchild domain

- Meso-scale phenomena affected by geography can be represented efficiently.
- Coupling with tidal flow simulation is the next step

Fluid-structure interaction between aquatic plants and water currents

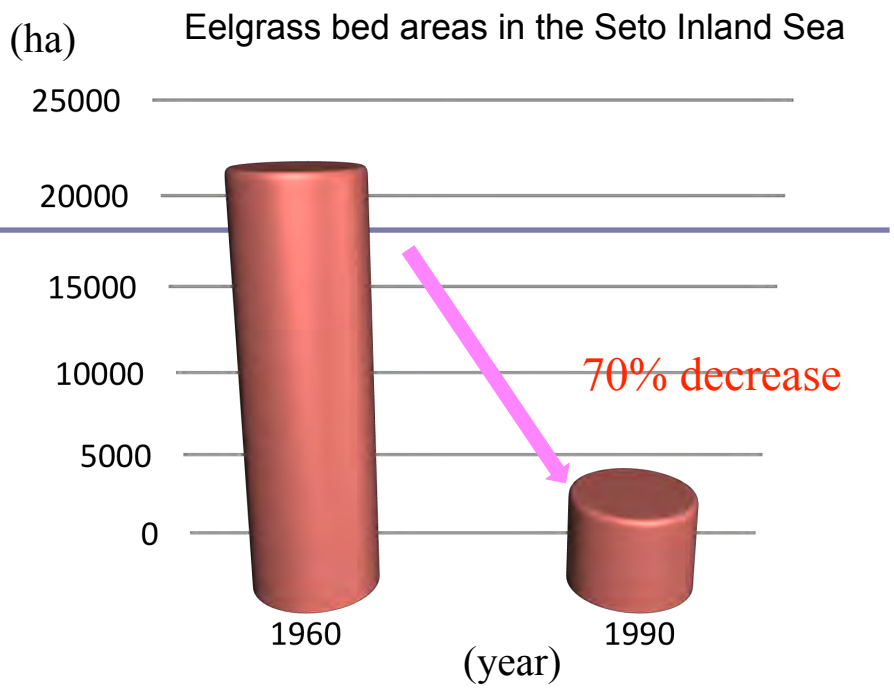


Background

Zostera marina

- The eelgrass grows in the ocean from the sandy shallow bottom. An area in which many eelgrass plants are found together is called an eelgrass beds.
- Eelgrass beds play an extremely important role in coastal ecosystems.
  - Supplying oxygen to the water by virtue of photosynthesis.
  - Providing an egg-laying sites for fish of many types, and servers other important purposes in marine environments.

Eelgrass bed



- Reclamation
- Shore protection

Decrease of eelgrass beds

Trials for recovery by artificial transplantation

The rates of eelgrass germination and fixation are not high.

A trade-off relation in water current speed

If the current is too fast,

Roots of eelgrass cannot stand against the high tension.

Eelgrass is swept away by the current.

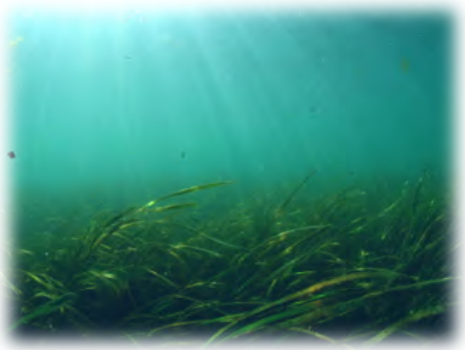
Fixation rate decrease.

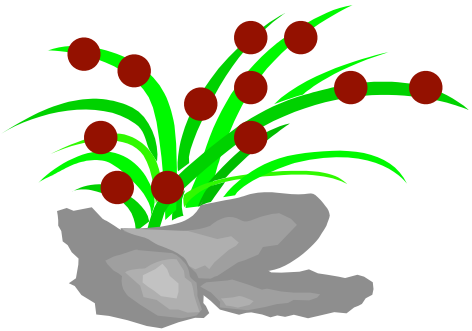
If the current is too slow,

The eelgrass leaves become covered with mud.

The eelgrass cannot photosynthesize enough.

Eelgrass dies.





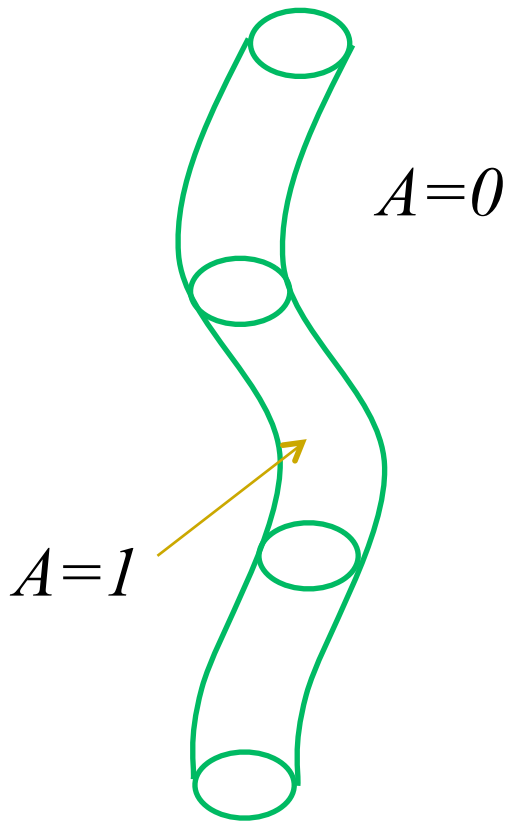
The flow situation of seawater strongly affects to the germination and fixation rates in artificial transplantations.

Water flow by immersed boundary method

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\,\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} - cA \cdot (\boldsymbol{u} - \boldsymbol{u}_a)$$

$$\nabla \cdot \boldsymbol{u} = 0$$

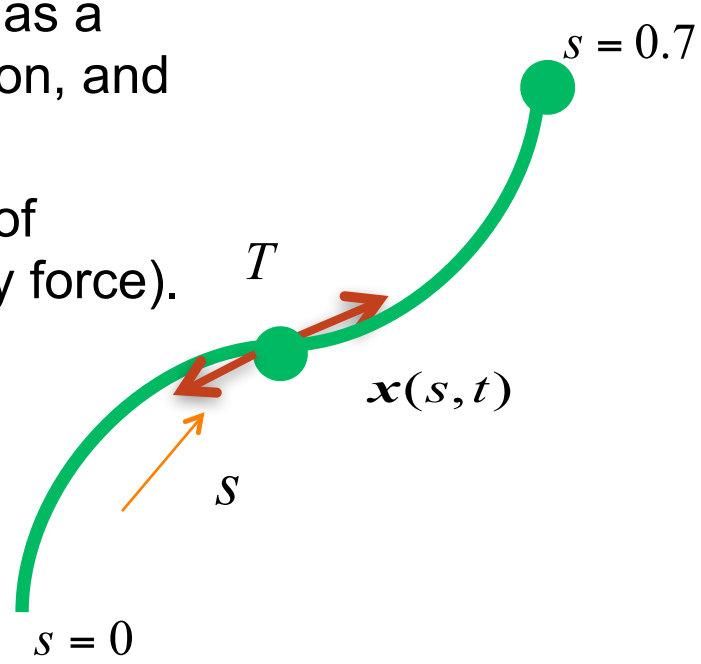
$$A\,(x, y, z, t) = \begin{cases} 1 & \text{in eelgrasses} \\ 0 & \text{out of eelgrasses} \end{cases}$$



- Navier-Stokes equation for incompressible viscous fluid is used where the existence of the eelgrass is included as a external force term that is proportional to the difference between the fluid and eelgrass velocities.
- Flow equation is solved on fixed Cartesian grid by using finite-difference approximation with staggered arrangement.

Motion equation for plants (structure)

- The motion equation of the eelgrass is represented as a balance of the acceleration, bending stiffness, tension, and external forces.
- The external force is computed through integration of pressure around the eelgrass and gravity (buoyancy force).



Governing equation

acceleration | bending stiffness | tension | external force

$$\rho_a \frac{\partial^2 \boldsymbol{x}}{\partial t^2} + EI \frac{\partial^4 \boldsymbol{x}}{\partial s^4} - \frac{\partial}{\partial s}\left(T \frac{\partial \boldsymbol{x}}{\partial s}\right) = \boldsymbol{F}$$

$$\left| \frac{\partial x}{\partial s} \right|^2 = 1$$

: Length of each segment should be constant.

ρ_a : Linear density
$\boldsymbol{x}$: Coordinate value
E : Young's modules
I : Geometrical moment of inertia
s : Position coordinate
T : Tension
$\boldsymbol{F}$: External force

Discretization

Discretize the governing equations with respect to time.

$$\rho_a \frac{\boldsymbol{x}^{n+1} - 2\boldsymbol{x}^n + \boldsymbol{x}^{n-1}}{\Delta t^2} + EI \frac{\partial^4 \boldsymbol{x}^{n+1}}{\partial s^4} - \frac{\partial}{\partial s}\left(T^{n+1} \frac{\partial \boldsymbol{x}^{n+1}}{\partial s}\right) = \boldsymbol{F}^n \qquad (1)$$

$$\left| \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \right|^2 = 1 \qquad (2)$$

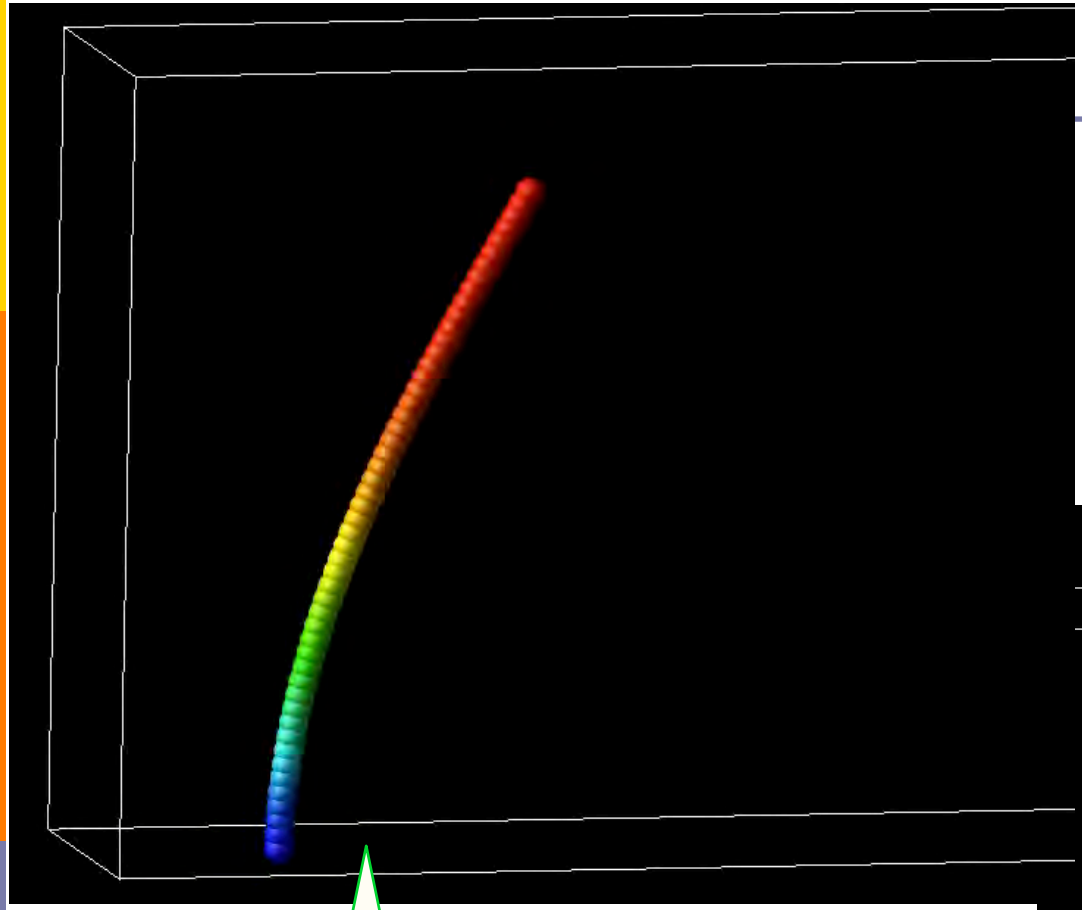
Take the divergence of (1) with respect to s, take inner-product with $\frac{\partial \boldsymbol{x}^{n+1}}{\partial s}$.
Then substitute (2) to it.

$$\frac{\partial^2 T^{n+1}}{\partial s^2} = \frac{\rho_a}{\Delta t^2}\left(1 - 2\frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \cdot \frac{\partial \boldsymbol{x}^n}{\partial s} + \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \cdot \frac{\partial \boldsymbol{x}^{n-1}}{\partial s}\right) - \frac{\partial \boldsymbol{F}^n}{\partial s} \cdot \frac{\partial \boldsymbol{x}^{n+1}}{\partial s} \qquad (3)$$

Solve (1) and (3) iteratively to obtain tension T and position $\boldsymbol{x}$ at $n+1$ time step.

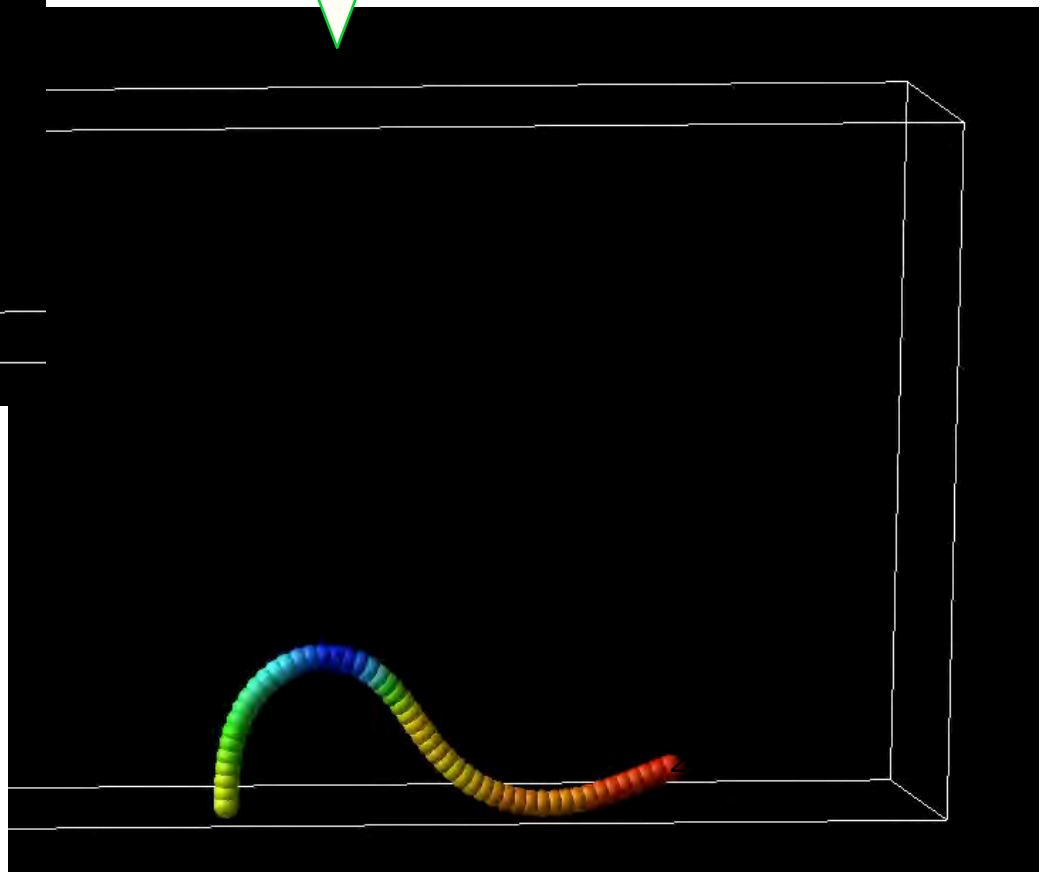
Tension plays a very similar role of pressure in incompressible flow computation.

Test calculations (eelgrass only)



Bending stiffness and buoyancy

The entire eelgrass plant rises because of the buoyancy effect.

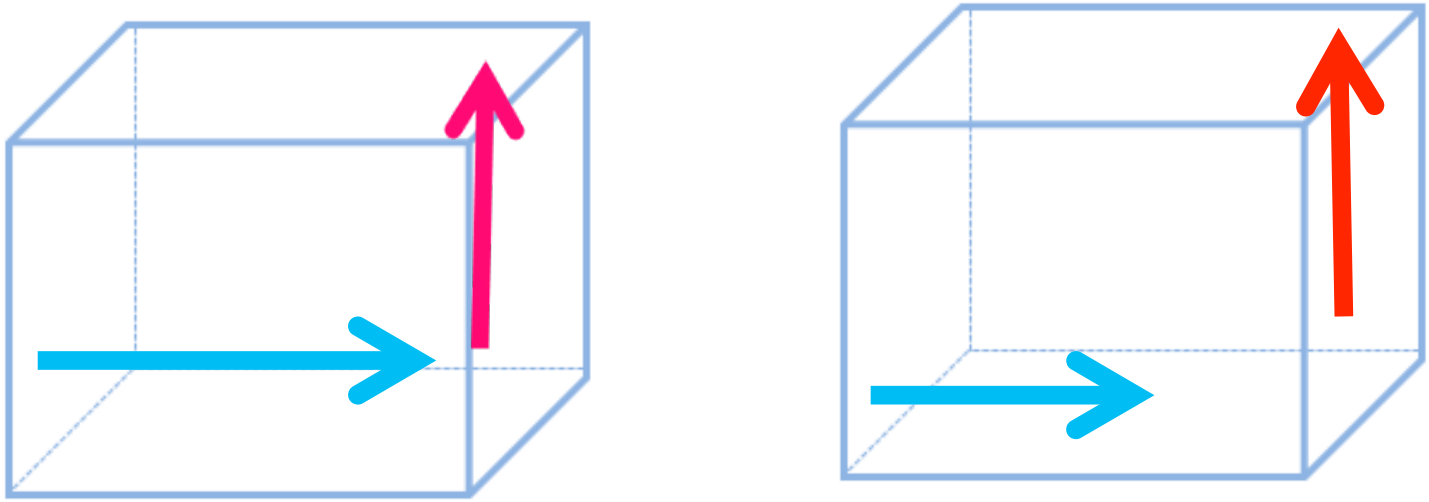


Bending stiffness force

The bent eelgrass becomes straight because of the effects of bending stiffness force.

Technical problems

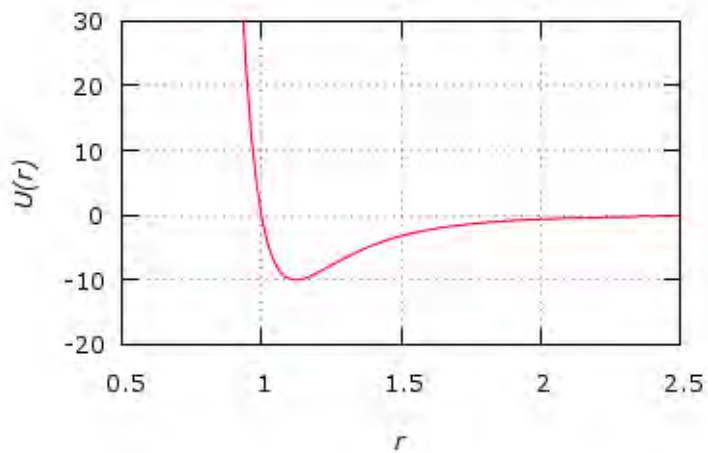
- In this study, repulsive force is used to prevent the eelgrass from passing through one another.



$$U(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

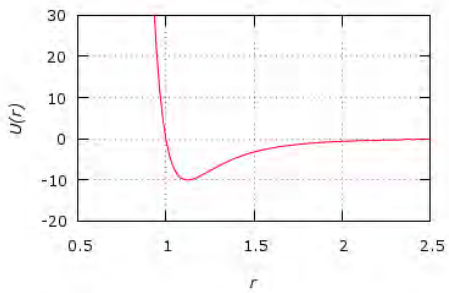
r : distance

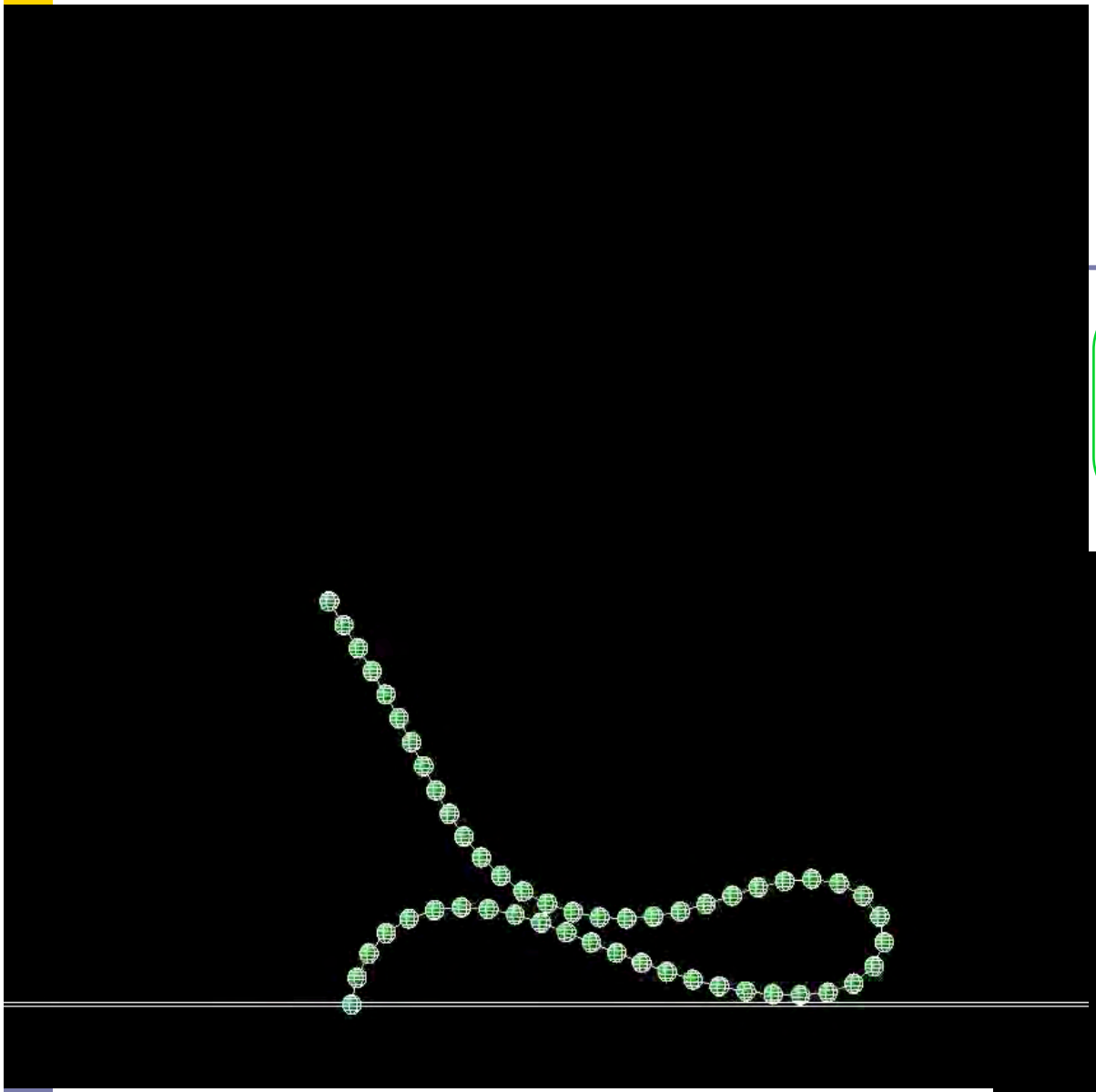
σ, ε : parameters

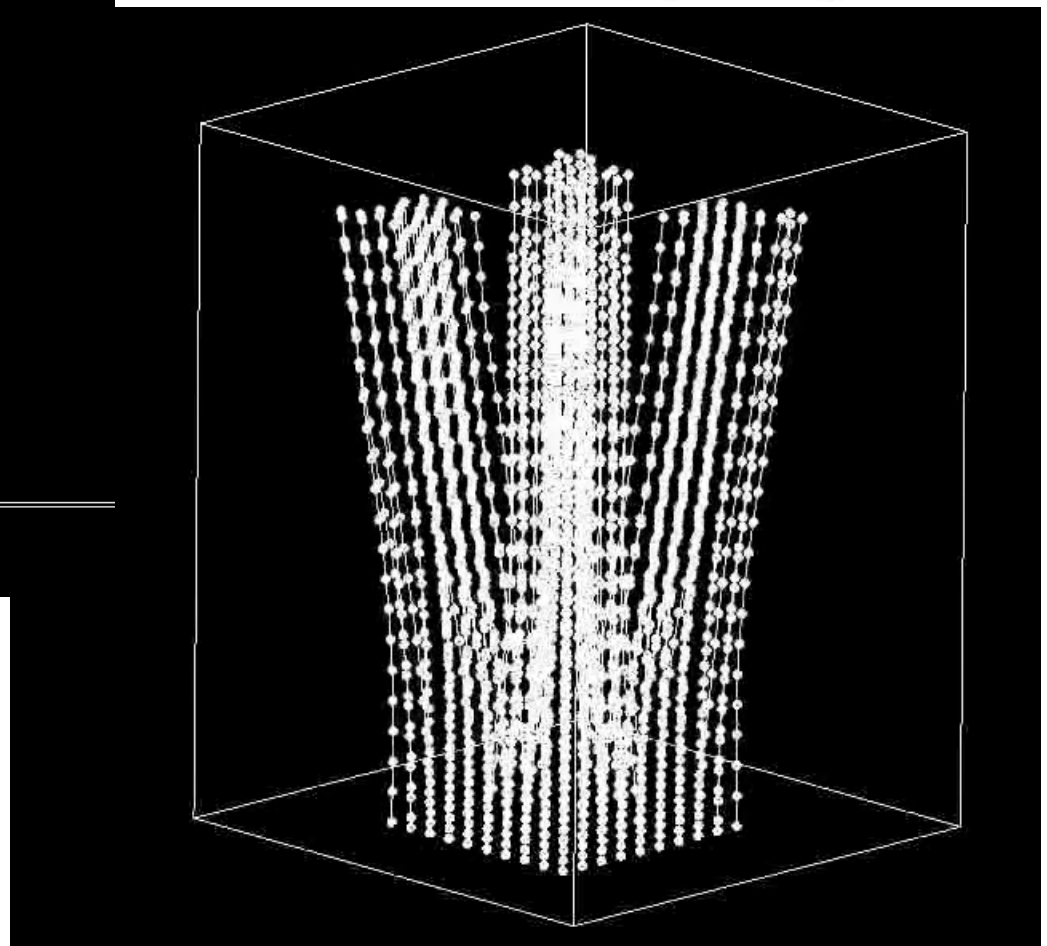


Test calculations for repulsive force

Unrealistically strong repulsive force is given here for checking purpose.



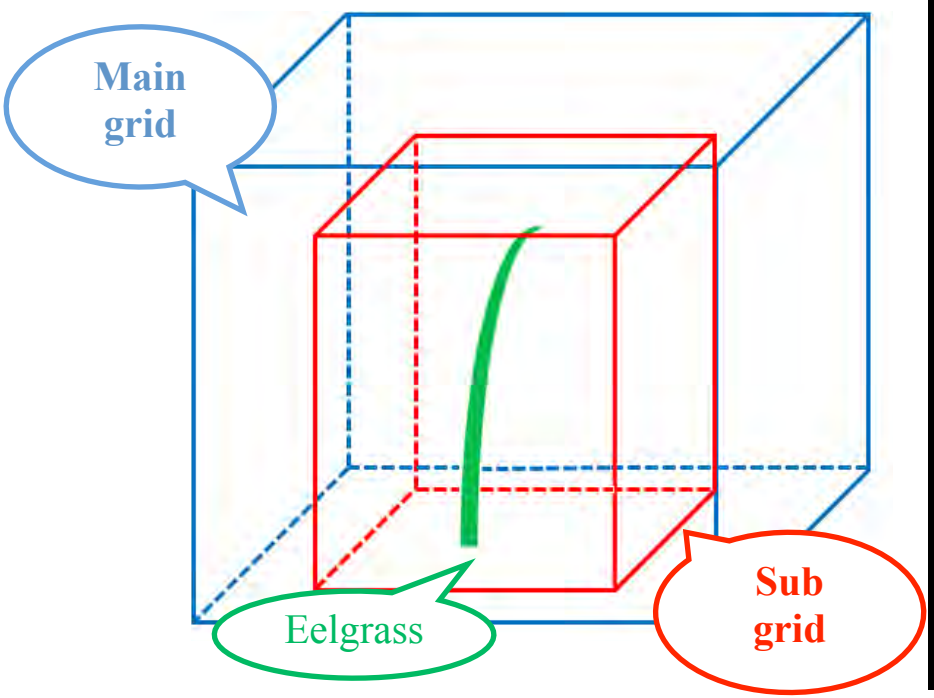


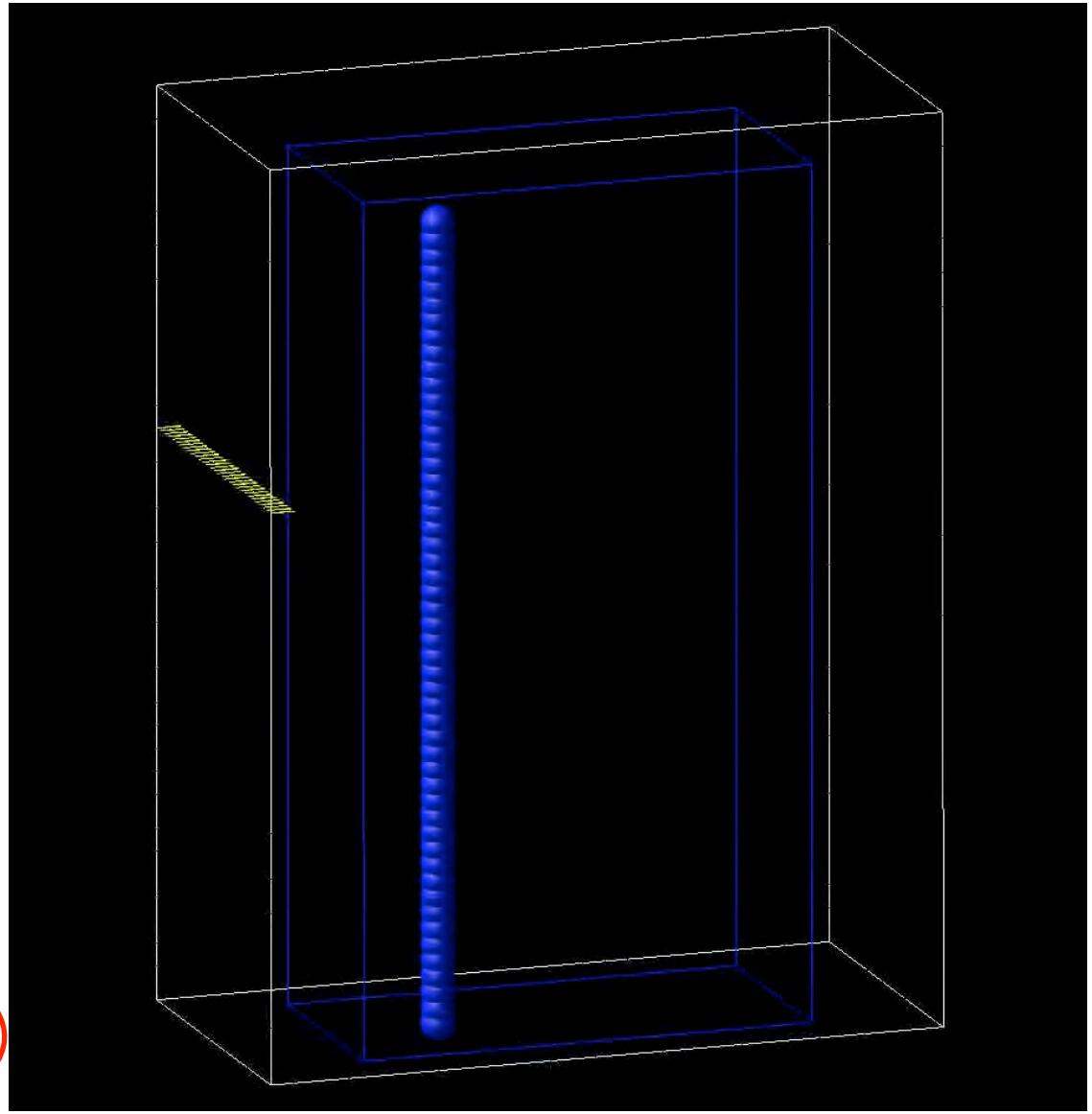


The eelgrass is prevented from passing through the bottom by the repulsion force between eelgrass and the bottom surface.

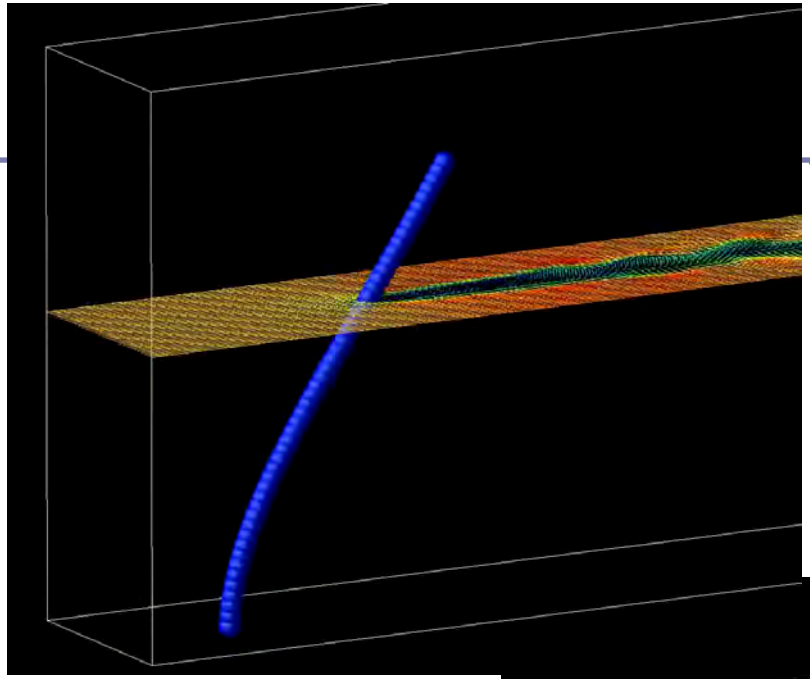
Overset grids for fluid

- In order to resolve the flow field near the eelgrass, overset grids have been applied.
- Finer fixed sub-grids are set around eelgrass.





Coupled simulation

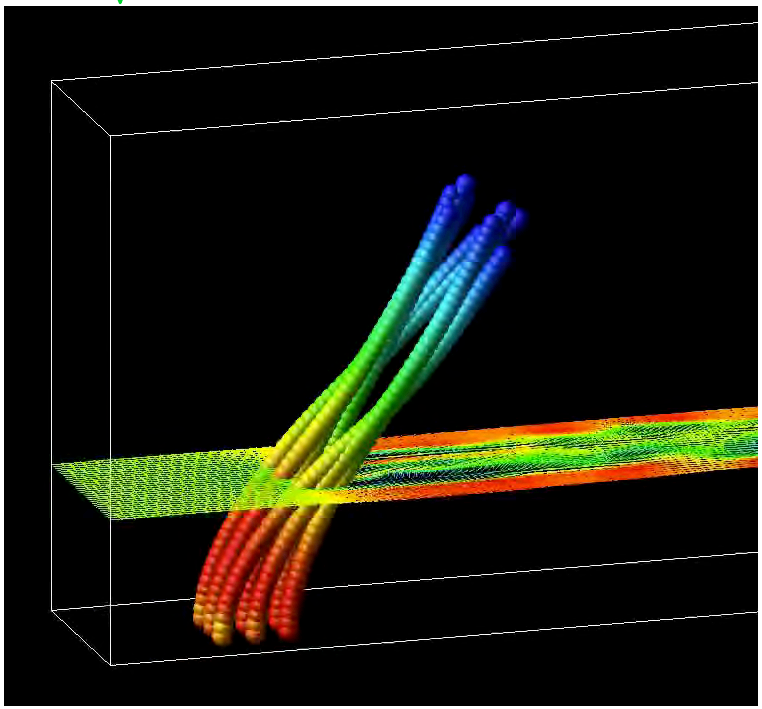


One eelgrass

The eelgrass is almost at rest when the vertical component of the force that the eelgrass receives from flow and the buoyancy are balanced.

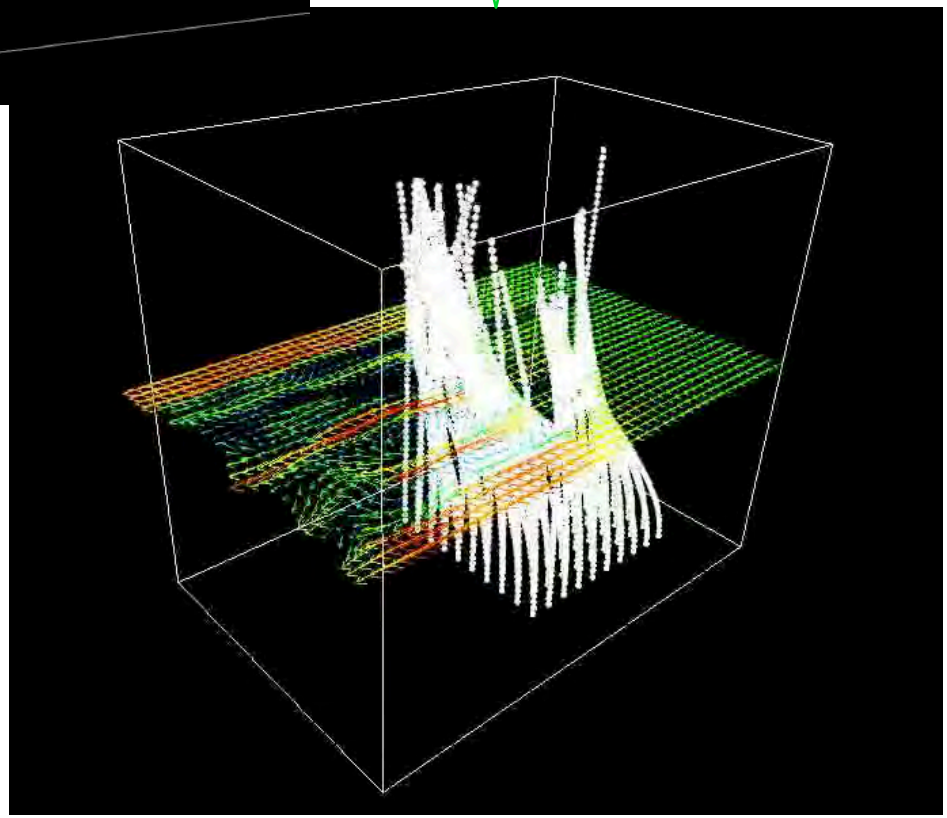
Nine eelgrass

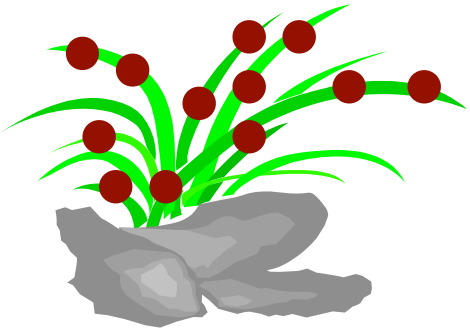
The eelgrass oscillates horizontally, because the flow field complexity increases as a result of the increasing number of eelgrass plants.



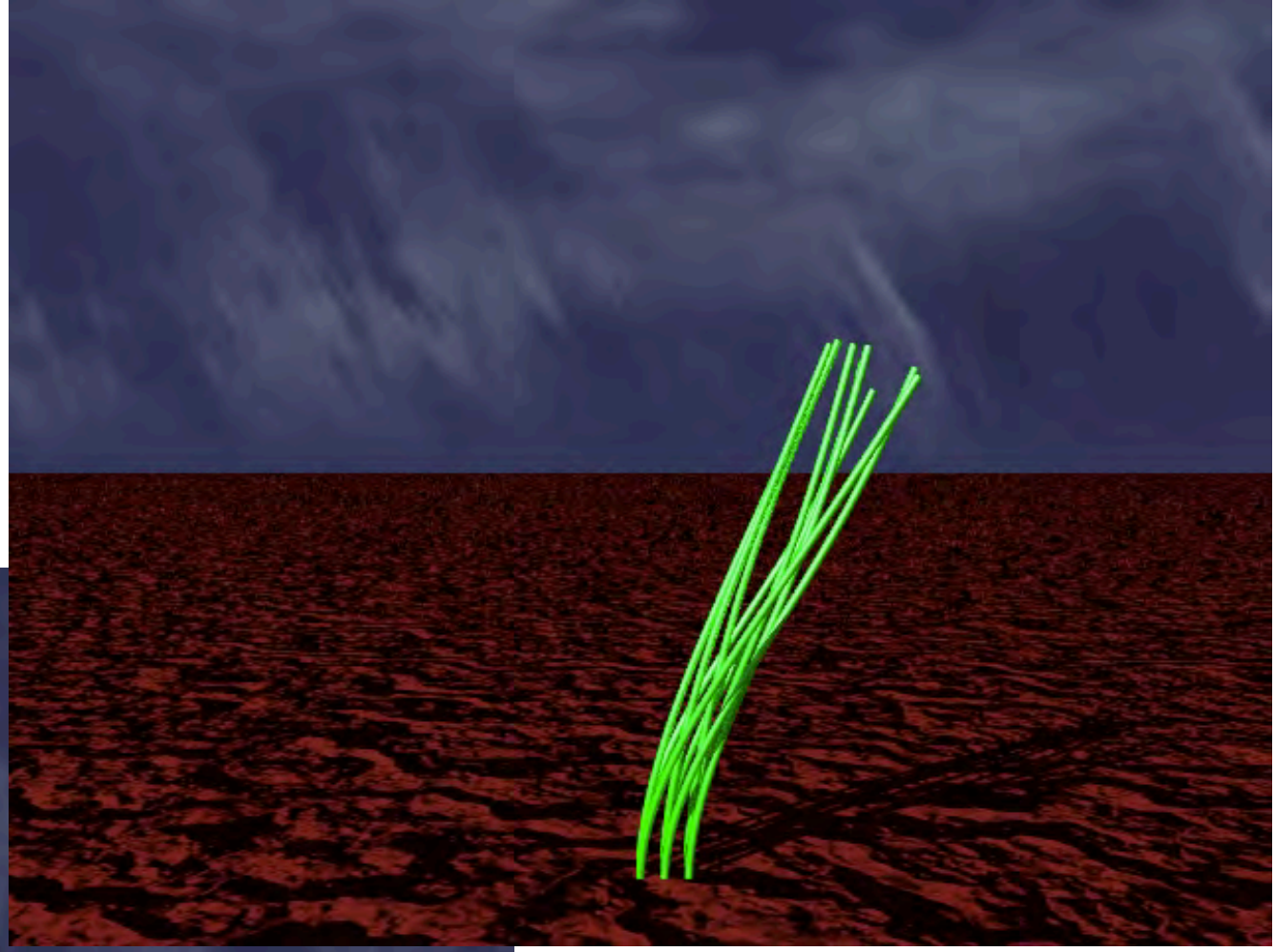
100 eelgrass

It is possible to conduct coupled simulation analyses in eelgrass bed.

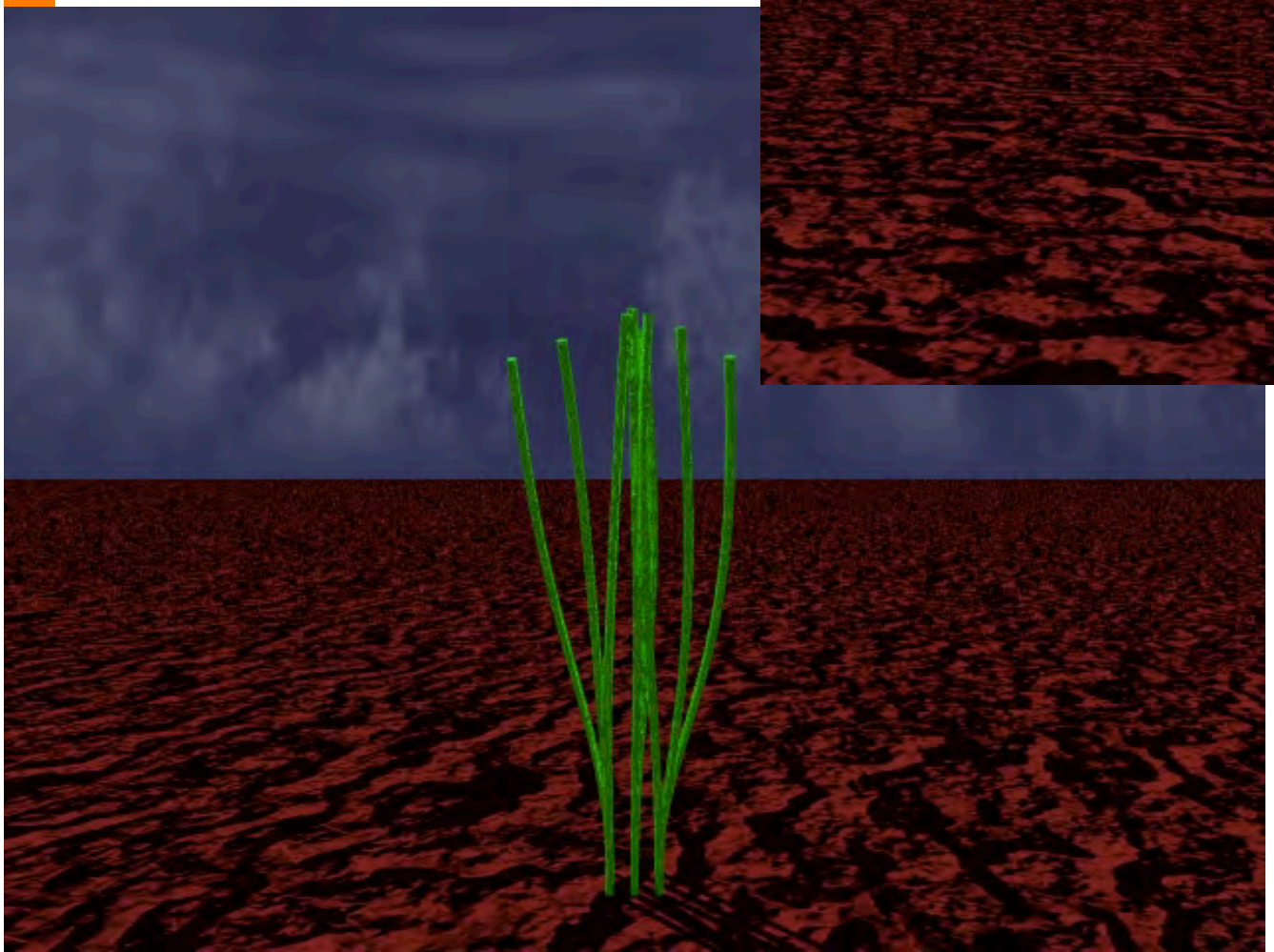




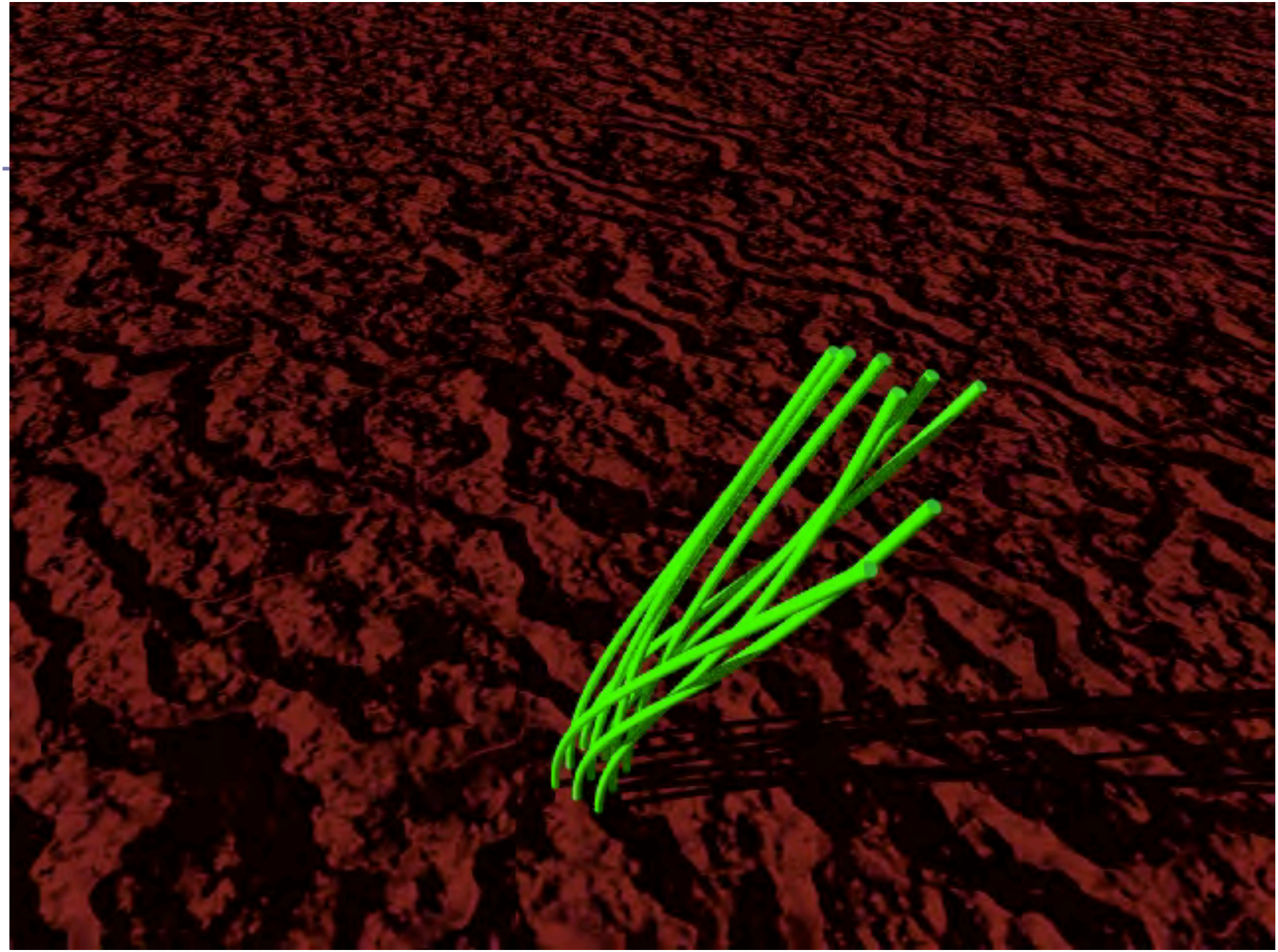
Back view

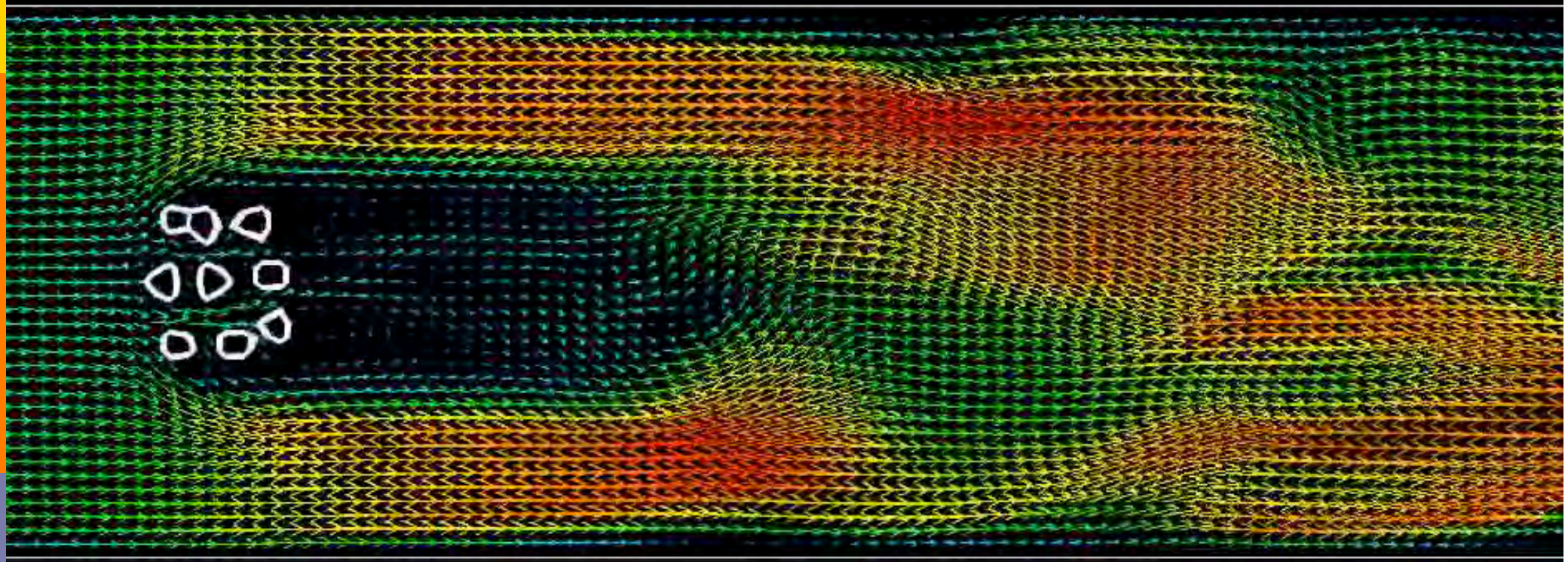


Side view

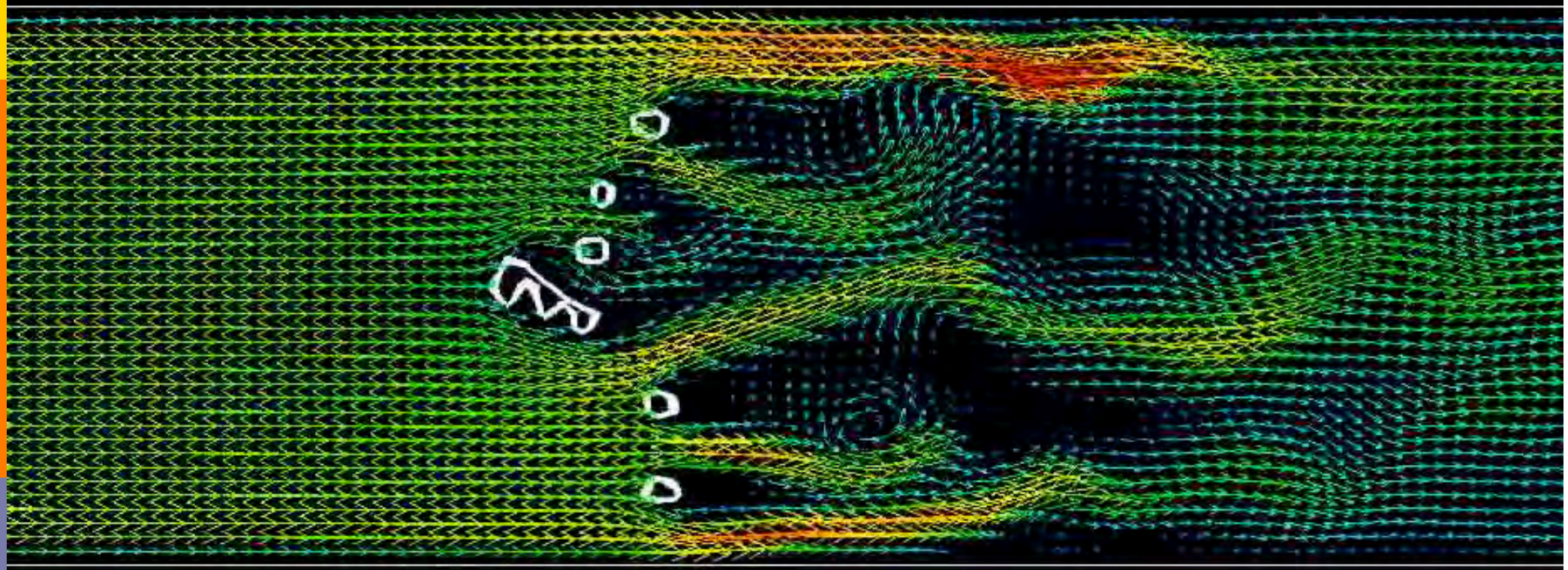


Fluttering motion is important from biological points of view, which enables eelgrass to shake the mud off its leaves.

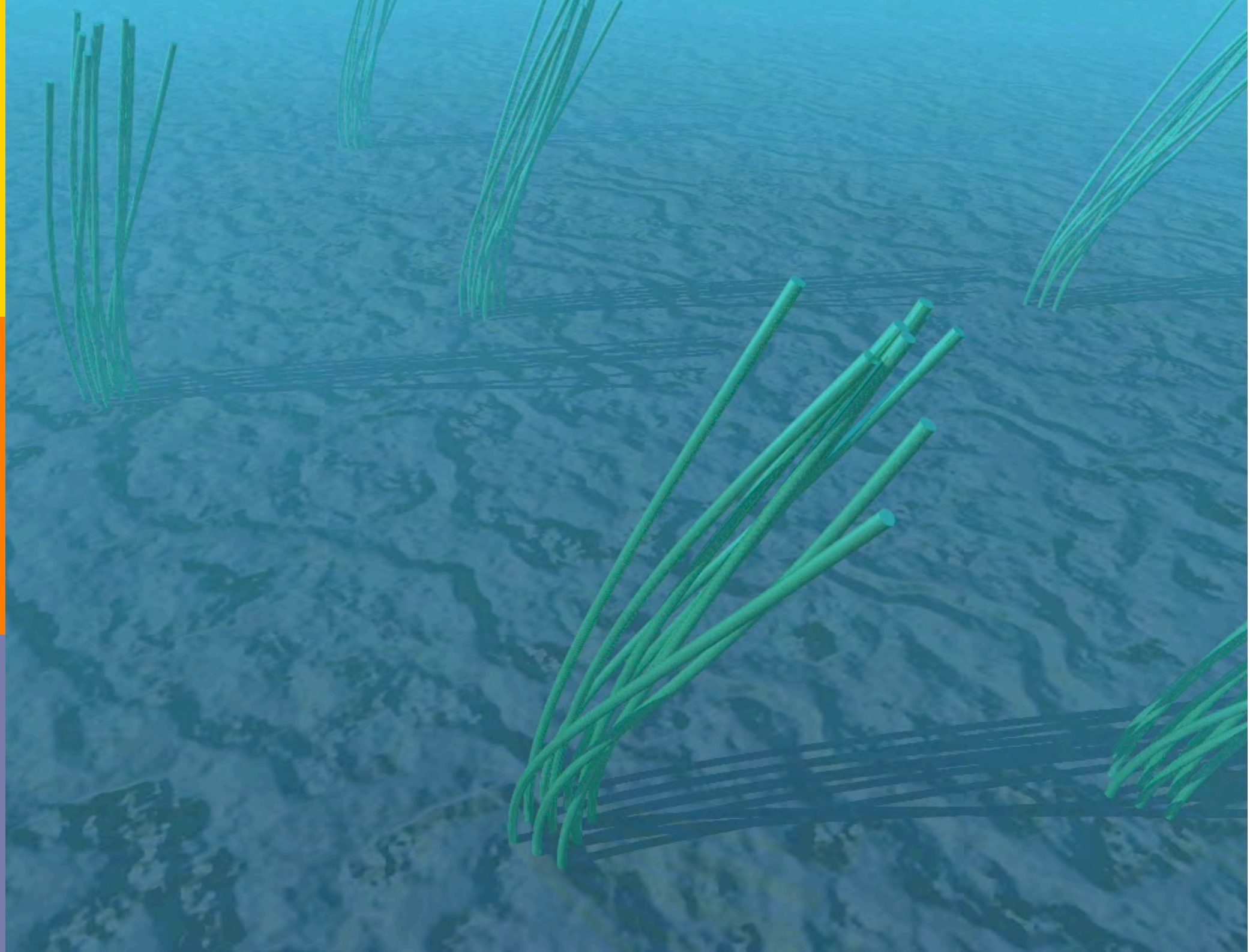




Horizontal cross section near the bottom

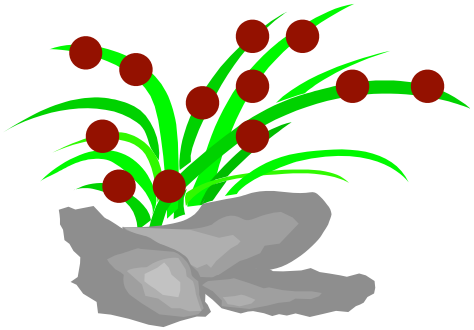


Horizontal cross section near water surface

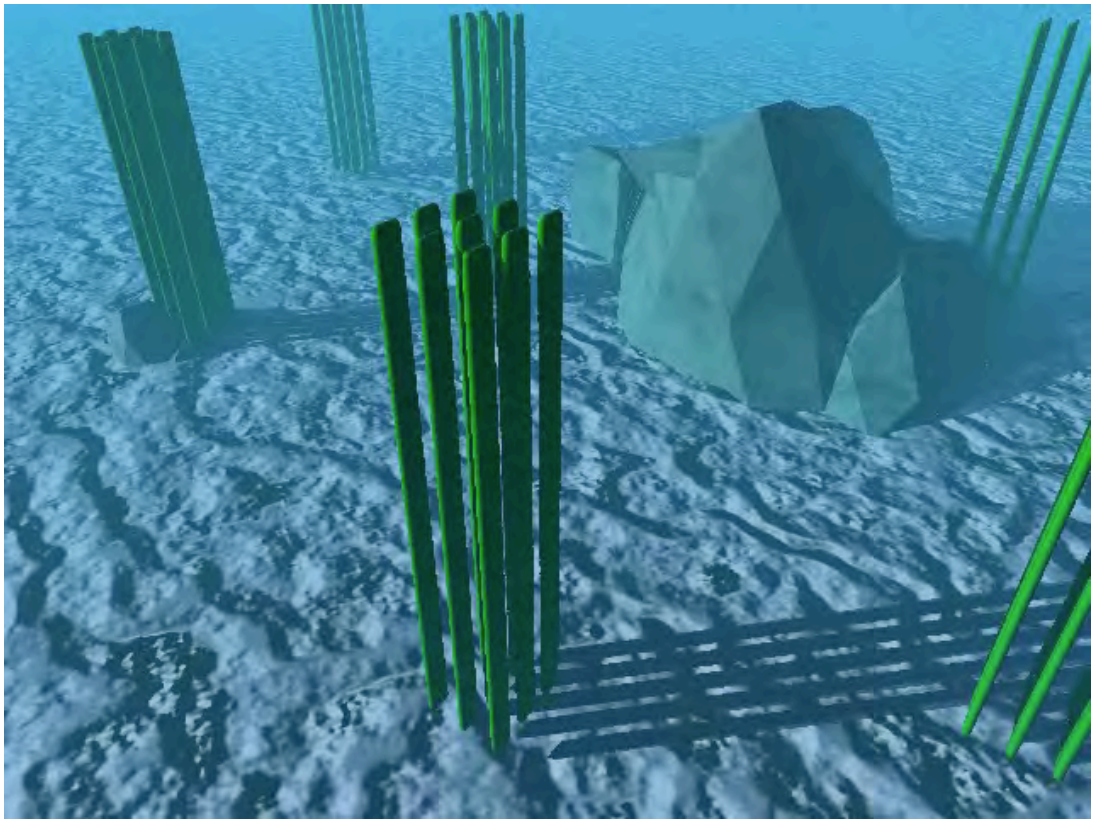


Conclusions and future works

- Fluttering motion of eelgrass is simulated by FSI.
- Fluttering motion is important from biological points of view, which enables eelgrass to shake the mud off its leaves.

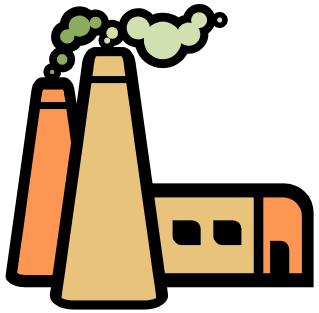


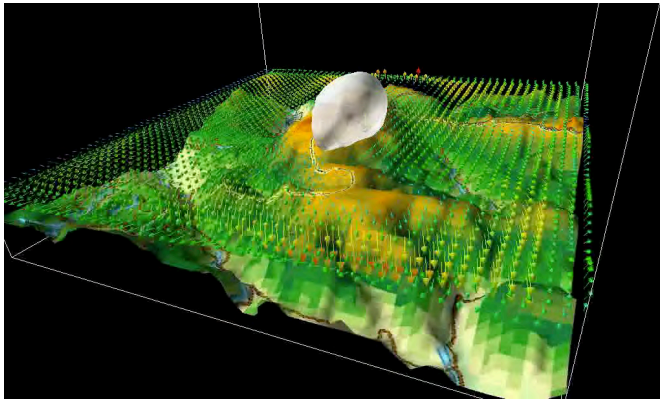
- More precise evaluation of force from fluid to solid is necessary.
- Modeling of motions of eelgrass as flow dampers or vortex/ disturbance generators in a macroscopic way, i.e., some kind of body forces on the flow is a reasonable next step.



Convection diffusion of toxic materials in mountainous area

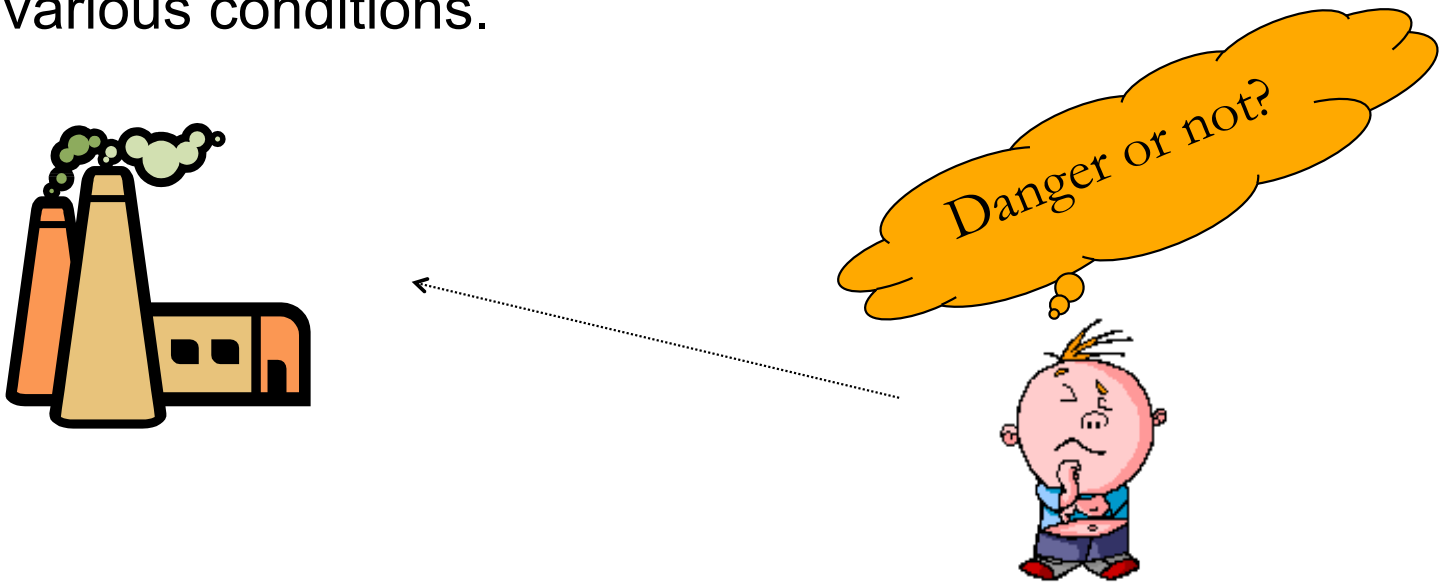
- A final disposal site for industrial wastes was planned to be constructed in a mountainous area surrounded by several small villages.
- Construction of an incinerator was added to the plan in order to decrease the bulk of wastes.
- The plan was completely legal and satisfied the environmental regulations. Local municipality approved the construction plan.





Convection diffusion of toxic materials in mountainous area

- Opposition campaign to the plan was started in one village near the incinerator site.
- The other villages on the other side of the mountain supported the construction plan, because they didn't think that the incinerator affected to their villages.
- Experiments were very difficult and un-realistic. Legal assessments were already completed.
- Then, the local municipality asked us to simulate the behavior of toxic materials under various conditions.



Convection diffusion of toxic materials in mountainous area

- Simulation cannot provide the final answer because there are a lot of conditions in the real situation.
- Simulations have shown a non-trivial possibility, which was not expected before.
- Mathematicians can contribute to provide “risk communication tools”.