## SUMMER SCHOOL on Control and numerics for fluid-structure interaction problems

WITHIN THE IFCAM PROJECT PDE CONTROL

BANGALORE, TIFR-CAM, JUNE 22-26, 2015

# Speakers of the Summer School

MIGUEL ÁNGEL FERNÁNDEZ Inria, miguel.fernandez@inria.fr Numerical approximation of fluid-structure systems

CÉLINE GRANDMONT Inria, celine.grandmont@inria.fr Modelling and Analysis of fluid-structure systems

MARIUS TUCSNAK Université de Lorraine, Marius.Tucsnak@univ-lorraine.fr Control of fluid-structure systems

# Organizing committee

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- Jean-Pierre Raymond, Université de Toulouse III
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### Sponsors

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# Lectures of the Summer School

### NUMERICAL APPROXIMATION OF FLUID-STRUCTURE INTERACTION PROBLEMS

MIGUEL ÁNGEL FERNÁNDEZ Inria, miguel.fernandez@inria.fr

Abstract. Mathematical problems describing the mechanical interaction of a flexible structure with an incompressible fluid flow appear in a wide variety of engineering fields. Fluid-structure interaction is also particularly ubiquitous in nature. One can think, for instance, of the wings of a bird interacting with the air, the fins of a fish moving through the water, or blood propelled into the arteries. The solid is deformed under the action of the fluid and the fluid flow is disturbed by the moving solid. Such multi-physic phenomena are generally described by heterogeneous systems of non-linear equations with an interface coupling which can be extremely stiff (the so-called addedmass effect). Over the last decade, the development and analysis of efficient numerical methods for these systems has been a very active field of research. The series of lectures is intended as an introduction to these techniques, with particular emphasis on spatial and time discretization (fitted/ unfitted meshes and coupling schemes), stability and accuracy.

#### Preliminary program :

- 1. Motivations
- 2. Model equations
  - \* Fluid and structure equations
  - \* Coupling conditions
  - \* Linear model problems
  - \* Global energy equality, variational setting
- 3. Spatial semi-discretization : finite element approximation
  - \* Fitted meshes : variational consistent load computation
  - \* Unfitted meshes : fictitious domain and XFEM Nitsche methods
- 4. Time-discretization : coupling schemes
  - \* Implicit coupling schemes
  - \* Explicit coupling schemes
  - \* The added-mass effect : an instability result
  - \* Semi-implicit coupling : splitting the geometry
  - \* Projection based semi-implicit coupling

- \* Stabilized explicit coupling : Robin-Robin schemes
- $\ast$  Robin-Neumann explicit coupling schemes

### 5. Concluding remarks

### References.

- M.A. Fernández. Coupling schemes for incompressible fluid-structure interaction : implicit, semi-implicit and explicit. SeMa Journal, 2011, pp. 59-108.

- M.A. Fernández. Incremental displacement-correction schemes for incompressible fluid-structure interaction : stability and convergence analysis. Numerische Mathematik, 2013, 123 (1), pp. 21-65.

#### MODELLING AND ANALYSIS OF FLUID-STRUCTURE SYSTEMS

#### CÉLINE GRANDMONT Inria & UPMC Paris, celine.grandmont@inria.fr

**Abstract.** Many physical phenomena deal with a fluid interacting with a moving or deformable structure. These kinds of problems have a lot of important applications, for instance, in areolasticity, biomechanics, hydroelasticity, sedimentation, etc. From the mathematical point of view they have been studied extensively over the past twenty years.

In this lecture, we will review some of these existence results on fluid-structure interaction problems. We will focus on viscous, Newtonian flow interacting with an elastic structure, in the case of large enough deformations of the structure inducing non negligible deformations of the fluid domain : the fluid equations are set in an unknown domain depending on the displacement of the elastic structure, itself resulting from the stress applied by the fluid. First we will present the coupled system and derived, at least formally, energy estimates. We will then explain the added mass effect phenomena which appears in these problem. Next on a specific model (a 2D fluid interacting with a 1D elastic or visco-elastic structure), we will review various existence results for the stationary problem or the full unsteady one. For each case, we will try to underline the difficulties and present various possible strategies to overcome them.

**References.** 1 - Céline Grandmont, Mária Lukáčová-Medvid'ova, Sárka Nečasová, Mathematical and Numerical Analysis of Some FSI Problems, Chapter 1, in 'Fluid-Structure Interaction and Biomedical Applications', Tomáš Bodnár, Giovanni P. Galdi, Šárka Nečasová Eds, Springer, 2014. http://link.springer.com/book/10.1007

### Analysis and Control of Some Particular Flows

### MARIUS TUCSNAK

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Abstract. In these lectures we consider systems coupling partial and ordinary differential equations which model the motion of solids in various type of fluids. These systems are subject to exterior inputs which act either on the fluid or only on the solid. We discuss, in particular, the case in which the control variable is the shape of the particle (the swimming problem). We focus on questions on controllability and stabilization of these coupled systems. Concerning he fluid, we consider a hierarchy of models, beginning with the one dimensional viscous Burgers equation, continuing with the equations of one dimensional viscous compressible fluids (the piston problem) and ending with the 3D incompressible Stokes and Navier-Stokes equations.

#### References.

[1] J.-M. Coron, Control and nonlinearity, Mathematical Surveys and Monographs, 136. American Mathematical Society, Providence, RI, 2007.

[1] Y. Liu, T. Takahashi and M. Tucsnak, Single input controllability of a simplified fluid-structure interaction model, ESAIM Control Optim. Calc. Var., 19 (2013), 20-42.

[2] J. Lohéac and M. Tucsnak, Controllability and time optimal control for low Reynolds numbers swimmers, Acta Appl. Math. 123 (2013), 175-200.

[3] M. Tucsnak and G. Weiss, Observation and control for operator semigroups, Birkhäuser Verlag, Basel, 2009. xii+483 pp.

Monday	Tuesday	Wednesday	Thursday	Friday
9h – 9h30 Registration Opening	9h30 – 11h MT	9h30 – 11h CG	9h30 – 11h MF	9h30 – 11h MT
9h30 – 10h30 CG 10h30- 11h Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11h  12h MF 12h 13h MT	11h30 – 13h CG	11h30 – 13h MF	11h30 – 13h MT	11h30 – 13h CG
Lunch	Lunch	Lunch	Lunch	Lunch
14h – 15h30 CG	14h – 15h30 MF	14h – 15h30 MT	14h – 15h30 CG	14h – 15h30 MF
Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
16h – 17h30 MF	16h – 17h30 MT	16h – 17h30 CG	16h – 17h30 MF	16h – 17h30 MT

CG : Céline Grandmont

MF : Miguel Fernandez

MT : Marius Tucsnak