Delta Waves and Explicit Formulae for Spherically Symmetric Solutions of the Multi-dimensional Zero-Pressure Gas Dynamics System

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In this talk, we intend to discuss a few recent results on the *multidimensional zero-pressure gas dynamics system*

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and the associated *adhesion model*

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Before proceeding further, a few quick remarks about a related system.

- Related to the system under discussion, we have the '*sticky particle dynamics*' model

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Let us begin with this result on the equations satisfied by the radial components of velocity and density, that is, when

$$u(x,t) = \frac{x}{r}q(r,t), \ \rho(x,t) = \rho(r,t), \ r = |x|$$

Theorem

The equations (1), for radial components, transform into the following system:

$$q_t + qq_r = 0,$$
$$\rho_t + r^{-(n-1)}(r^{(n-1)}\rho q)_r = 0.$$

Using the transformation $\rho(r,t) = r^{-(n-1)}p(r,t)$, the second equation above can be written as

$$p_t + (pq)_r = 0.$$

Let us recall that

Definition

A family of smooth functions $(u^\epsilon, \rho^\epsilon)_{\epsilon>0}$ is called a *weak asymptotic solution* of the system (1) with initial conditions $u(x,0) = u_0(x)$, $\rho(x,0) = \rho_0(x)$ provided the relations

$$\begin{aligned} u_t^\epsilon + (u^\epsilon.\nabla)u^\epsilon &= o_{\mathfrak{D}'}(\mathbb{R}^n)(1), \\ \rho_t^\epsilon + \nabla.(\rho^\epsilon u^\epsilon) &= o_{\mathfrak{D}'}(\mathbb{R}^n)(1), \\ u^\epsilon(x,0) - u_0(x) &= o_{\mathfrak{D}'}(\mathbb{R}^n)(1), \\ \rho^\epsilon(x,0) - \rho_0(x) &= o_{\mathfrak{D}'}(\mathbb{R}^n)(1) \end{aligned}$$

hold uniformly in $t > 0$.

Explicit weak asymptotic solutions

We then have the following result on explicit weak asymptotic solution of the system (1) using the adhesion approximation (2), for gradient velocities.

Theorem

Assume $u_0(x) = \nabla_x \phi_0$ where $\phi_0 \in W^{1,\infty}(R^n)$ and $\rho_0 \in L^\infty(R^n)$. Let $\phi_0^\epsilon = \phi_0 * \eta^\epsilon$, $\nabla_x \phi_0^\epsilon = \nabla_x(\phi_0 * \eta^\epsilon)$ and $\rho_0 = \rho_0 * \eta^\epsilon$, where η^ϵ is the usual Friedrichs mollifier in the space variable $x \in R^n$. Further let

$$u^\epsilon(x,t) = \frac{\int_{R^n} (\nabla_y \phi_0^\epsilon(y)) e^{\frac{-1}{\epsilon}\left[\frac{|x-y|^2}{2t} + \phi_0^\epsilon(y)\right]} dy}{\int_{R^n} e^{\frac{-1}{\epsilon}\left[\frac{|x-y|^2}{2t} + \phi_0^\epsilon(y)\right]} dy}, \tag{4}$$

$$\rho^\epsilon(x,t) = \rho_0^\epsilon(X^\epsilon(x,t,0)) J^\epsilon(x,t,0),$$

where $X^\epsilon(x,t,s)$ is the solution of $\frac{dX^\epsilon(s)}{ds} = u^\epsilon(X^\epsilon, s)$ with $X^\epsilon(s=t) = x$ and $J^\epsilon(x,t,0)$ is the Jacobian matrix of $X^\epsilon(x,t,0)$ w.r.t. x. Then $(u^\epsilon, \rho^\epsilon)$ is a weak asymptotic solution to (1).

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An interesting consequence

When the initial velocity $u_0(x) = \frac{x}{r} q_0(r)$, it can be written in gradient form

$$u_0(x) = \nabla_x \phi_0(x) = \nabla_x \Big(\int_0^{|x|} q_0(s)\ ds \Big)$$

and hence the previous result applies.

Furthermore we can deduce the following asymptotic behaviour:

- With initial data satisfying $\int_0^{\infty} q_0(s)\ ds < \infty$ and $\int_{\mathbb{R}^n} \rho_0(x)\ dx < \infty$, the velocity component goes to 0 as t tends to ∞ uniformly on compact subsets of $\mathbb{R}^n$ and the mass $\int_{\mathbb{R}^n} \rho(x,t)\ dx$ is conserved.

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Explicit radial solutions for the inviscid system

We next move on to the construction of explicit global radial solution of

$$u_t + (u.\nabla)u = 0,\ \rho_t + \nabla.(\rho u) = 0,$$

with initial conditions

$$u(x,0) = \frac{x}{r}q_0(r),\ \rho(x,0) = \rho_0(r) = r^{-(n-1)}p_0(r),\ r = |x|,$$

a condition on the mass

$$\int_{\mathbb{R}^n} \rho(x,t)\ dx = p_B(t)$$

and prescribed normal velocity at the origin

$$\lim_{x\to 0} \frac{x}{r}u(x,t) = q_B(t). \tag{5}$$

- $q_0(r)$ and $p_0(r)$ are considered to be in BV and the functions p_B, q_B are considered to be continuous.

We recall that q and $p = r^{(n-1)}\rho$ satisfy the equations

$$q_t + qq_r = 0, \ \ p_t + (pq)_r = 0.$$

We supplement them with the initial conditions $q_0(r), \ p_0(r)$ and a boundary condition for q and an integral condition on p:

$$q(0, t) = q_B(t), \ \omega_{n-1} \int_0^\infty p(r, t) \ dr = p_B(t).$$

The boundary and integral conditions have to be understood in a weak sense, that is, for q we have the condition

$$\text{either } q(0+, t) = q_B(t)$$

$$\text{or, } q(0+, t) \leq 0 \text{ and } q^2(0+, t) \leq q_B^+(t)^2$$

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We use the Hopf-Lax type formula for scalar conservation laws posed in a quarter plane proved in

- K.T.Joseph, Burgers' equation in the quarter plane, a formula for the weak limit, *Comm. Pure Appl. Math.*, **41**(1988) 133-149

and further developed in

- K.T.Joseph and G.D.Veerappa Gowda, Explicit formula for the solution of convex conservation laws with boundary condition, *Duke Math Jl.*, **62** (1991) 401-416.

In this context, we would also like to mention about an alternate formulation proved in

- P.G.Lefloch, Explicit formula for scalar nonlinear conservation laws with boundary condition, *Math. Methods Appl. Sci.*, **10**(1988), no.3, 265-287.

We introduce a class of paths in the quarter plane D.

For each fixed $(r, r_0, t),\ r > 0,\ r_0 \geq 0,\ t > 0$, let $C(r, r_0, t)$ denote the following class of paths β:

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- Each path is connected from the point $(r_0, 0)$ to (r, t) and is of the form $z = \beta(s)$, where β is a piecewise linear function of maximum three lines.
- Let β_0 denote the straight line path connecting $(r_0, 0)$ and (r, t) which does not touch the space boundary.

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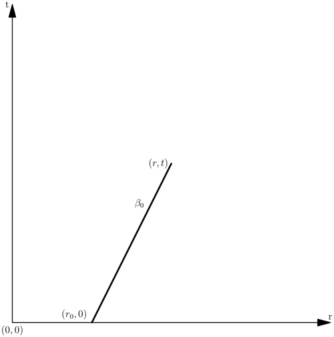
t
(r,t)
β_0
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r
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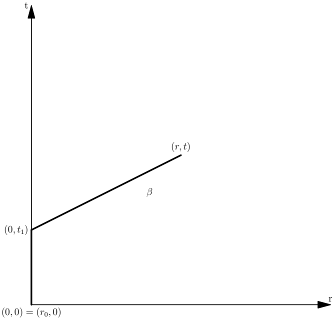
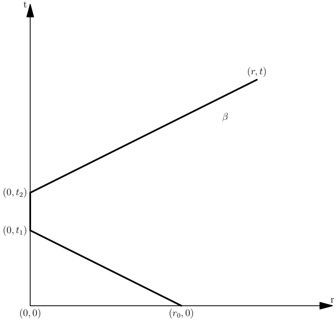

t
(r,t)
β
$(0,t_1)$
r
$(0,0)=(r_0,0)$


t
(r,t)
β
$(0,t_2)$
$(0,t_1)$
r
$(0,0)$
$(r_0,0)$

On $C(r, r_0, t)$, we define a functional

$$J(\beta) = -\frac{1}{2}\int_{\{s:\beta(s)=0\}} (q_B(s)^+)^2 \; ds + \frac{1}{2}\int_{\{s:\beta(s)\neq 0\}} (\frac{d\beta(s)}{ds})^2 \; ds \tag{6}$$

Let

$$A(r, r_0, t) = J(\beta_0) = \frac{(r - r_0)^2}{2t}.$$

For any $\beta \in C^*(r, r_0, t) = C(r, r_0, t) - \beta_0$ made up of three pieces, namely lines joining $(r_0, 0)$ to $(0, t_1)$ in the interior, $(0, t_1)$ to $(0, t_2)$ on the boundary and $(0, t_2)$ to (r, t) in the interior, it can be easily seen from (6) that

$$J(\beta) = J(r, r_0, t, t_1, t_2) = -\int_{t_1}^{t_2} \frac{(q_B(s)^+)^2}{2} \; ds + \frac{r_0^2}{2t_1} + \frac{r^2}{2(t - t_2)}.$$

For curves $\beta \in C^*(r, r_0, t)$ made up of two straight lines with one piece lying on the boundary $r = 0$, a similar expression can be written down.

It can then be proved that there exists $\beta^* \in C^*(r, r_0, t)$ and corresponding $t_1(r, r_0, t), t_2(r, r_0, t)$ so that

$$\begin{aligned} B(r, r_0, t) &= \min\{J(\beta) : \beta \in C^*(r, r_0, t)\} \\ &= \min\{J(r, r_0, t, t_1, t_2) : \; 0 \leq t_1 < t_2 < t\} \\ &= J(r, r_0, t, t_1(r, r_0, t), t_2(r, r_0, t)). \end{aligned}$$

- The function $B(r, r_0, t)$ is Lipschitz continuous.

Let

$$\begin{aligned} m(r, r_0, t) &= \min\{J(\beta) : \beta \in C(r, r_0, t)\} \\ &= \min\{A(r, r_0, t), B(r, r_0, t)\} \end{aligned}$$

and

$$Q(r, t) = \min_{r_0 \geq 0}\{m(r, r_0, t) + \int_0^r q_0(s)ds\}$$

- The functions $m(r, r_0, t)$ and $Q(r, t)$ are Lipschitz continuous.
- The minimum in $Q(r, t)$ is attained at some value of $r_0 \geq 0$, which we denote by $r_0(r, t)$.

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- The minimum in $Q(r, t)$ is attained at some value of $r_0 \geq 0$, which we denote by $r_0(r, t)$.

With these notations, we have the following result.

Theorem

With $r_0(r,t), A(r, r_0(r,t), t), B(r, r_0(r,t), t), t_1(r,t), t_2(r,t)$ as defined above, define

$$u(x,t) = \frac{x}{r}\begin{cases} \frac{r-r_0(r,t)}{t}, & \text{if } A(r, r_0(r,t), t) < B(r, r_0(r,t), t), \\ \frac{r}{t-t_1(x,t)}, & \text{if } A(r, r_0(r,t), t) > B(r, r_0(r,t), t), \end{cases} \tag{7}$$

and

$$P(r,t) = \begin{cases} -\int_{r_0(r,t)}^{\infty} p_0(z)dz, & \text{if } A(r, r_0(r,t), t) < B(r, r_0(r,t), t), \\ \omega_{n-1} p_B(t_2(x,t)), & \text{if } A(r, r_0(r,t), t) > B(r, r_0(r,t), t). \end{cases} \tag{8}$$

and set

$$\rho(x,t) = \frac{\partial_r(P(r,t))}{r^{(n-1)}}. \tag{9}$$

Then the distribution $(u(x,t), \rho(x,t))$ given by (7)-(9) satisfies (1). Further it satisfies the initial conditions, mass conditions and normal velocity at the origin (5) in the weak sense as described before.

Generalized Rankine-Hugoniot conditions

- It was shown by Albeverio and Shelkovich that a *generalized $\delta-$shock wave type solution* of (1) of the form

$$u(x,t) = u_+(x,t)\ H(S(x,t)>0) + u_-(x,t)\ H(S(x,t)<0),$$
$$\rho(x,t) = \bar{\rho}_+(x,t)\ H(S(x,t)>0) + \bar{\rho}_-(x,t)\ H(S(x,t)<0) + \hat{e}(t)\ \delta_{S(x,t)=0},$$

  where u_+,u_-,$\bar{\rho}_+$ and $\bar{\rho}_-$ are smooth functions away from the surface $S(x,t)=0$, satisfies the *generalized Rankine-Hugoniot conditions*

$$S_t + u_\delta.\nabla_x S|_{\Gamma_t} = 0,$$
$$\frac{\delta \hat{e}}{\delta t} + \nabla_{\Gamma_t}(\hat{e}u_\delta) = ([\bar{\rho}u] - [\bar{\rho}]u_\delta).\nabla_x S|_{\Gamma_t},$$

  along the surface of discontinuity $S(x,t)=0$, where $u_\delta = \frac{u_+ + u_-}{2}$ and $\Gamma_t = \{x : S(x,t) = 0\}$.
- The explicit solutions that we obtain for the radial case satisfy the generalized Rankine-Hugoniot conditions stated above.

References

The topics discussed in this lecture can be found in the following two articles:

- Anupam Pal Choudhury, K.T.Joseph, Manas R. Sahoo, *Spherically symmetric solutions of multidimensional zero-pressure gas dynamics system*, Journal of Hyperbolic Differential Equations, Vol. 11, Issue 2, June 2014.
- Anupam Pal Choudhury, K.T.Joseph, *Product of distributions and the zero-pressure gas dynamics system*, Sao Paulo Journal of Mathematical Sciences 7, 2(2013), 253-277.

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- I would like to thank *Prof. M.Vanninathan* for being an incessant source of inspiration.
- I would also like to take this opportunity to thank my *family* for their love and patience.

Thank You