

# Nonexistence of $k$ -peak solutions to elliptic problems in convex domains

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ICM Satellite Conference on PDE and Related Topics  
TIFR-CAM Bangalore, India 13-17 August 2010

## The Theorem of Gidas, Ni and Nirenberg (1979)

Let us consider a solution  $u \in C^2(\Omega) \cap C(\bar{\Omega})$  of the problem

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where  $\Omega \subset \mathbf{R}^N$  is a bounded domain which is convex with respect to  $x_1, \dots, x_N$  and is symmetric with respect to the planes  $x_1 = 0, \dots, x_N = 0$  and  $f$  is a locally Lipschitz function.

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## Corollary

A consequence of the previous theorem is that 0 is the unique critical point of  $u$ .

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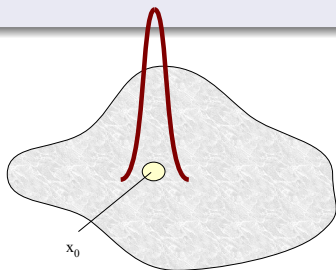
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### Conjecture

Let  $u$  be a solution of

$$\begin{cases} -\Delta u = u^p & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

with  $1 < p < \frac{N+2}{N-2}$ . Then, if  $\Omega$  is (strictly) convex, the monotonicity property holds.

## A brief history on the problem

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### Cabré and Chanillo (1998)

Let  $\Omega$  be a bounded, convex domain of  $\mathbf{R}^2$  where the boundary has positive curvature. Suppose that  $f \in C^\infty(\mathbf{R})$ ,  $f \geq 0$  and let  $u$  be a semi-stable solution of (i.e. the first eigenvalue of the linearized operator is nonnegative)

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Then  $u$  has a unique critical point  $x_0$  in  $\Omega$ .

### Applications

The previous result applies to the minimal solutions to  $f(s) = \lambda e^s$ ,  $\lambda > 0$ , and  $f(s) = \lambda(1+s)^p$ ,  $\lambda > 0$  and  $p > 1$ .

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### Grossi and Molle (2003)

Let  $\Omega$  be a bounded, convex domain of  $\mathbf{R}^N$ ,  $N \geq 3$ . Let us suppose that  $u_\epsilon$  is a positive solution of

$$\begin{cases} -\Delta u = u^{\frac{N+2}{N-2}-\epsilon} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

which satisfies ( $S$  is the best constant in Sobolev embedding)

$$\frac{\int_{\Omega} |\nabla u_\epsilon|^2}{\left( \int_{\Omega} |u_\epsilon|^{\frac{2N}{N-2}-\epsilon} \right)^{\frac{2(N+2)}{2N+\epsilon(N+2)}}} \rightarrow S$$

Then, denoting by  $x_\epsilon$  the point where the maximum of  $u_\epsilon$  is achieved, we have that  $(x - x_\epsilon) \cdot \nabla u_\epsilon(x) < 0$  for any  $x \in \Omega \setminus \{x_\epsilon\}$ , provided  $\epsilon$  is small enough.

## A problem in the plane: the case of one peak

Let us consider the following "model problem",

### The Gelfand problem

$$\begin{cases} -\Delta u = \lambda e^u & \text{in } \Omega \subset \mathbf{R}^2, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

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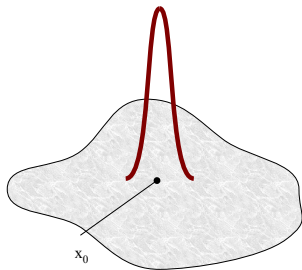
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### Theorem

For any  $\Omega \subset \mathbb{R}^2$ , there exists a solution  $u_\lambda$  with one peak, i.e.

$$u_\lambda(x) \rightarrow G(x, x_0) \quad \text{as } \lambda \rightarrow 0.$$

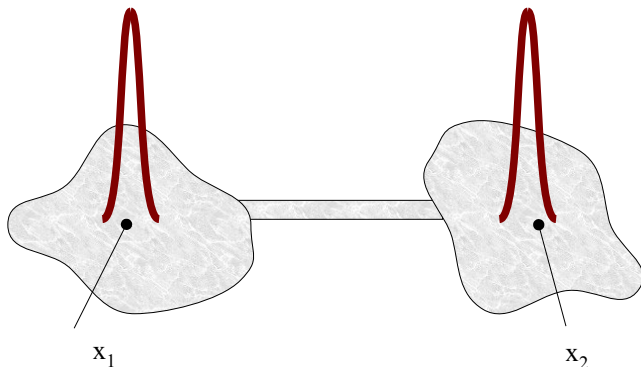


## A problem in the plane: the case of $k$ peaks.

According to the shape of the domain, we can have solution with more peaks

### Theorem, Esposito, Grossi e Pistoia (2005)

Under some suitable assumptions on  $\Omega \subset \mathbf{R}^2$ , there exist solution with  $k$  peaks, for any  $k \geq 2$ .



The location of the peaks of the solution has been characterized.

Let us denote by  $P_{1,\lambda} \rightarrow P_1, \dots, P_{k,\lambda} \rightarrow P_k$  the peaks of the solution  $u_\lambda$ .

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Recall that  $G(x, y) = -\frac{1}{2\pi} \log|x - y| - H(x, y)$  and  $R(x) = H(x, x)$ .

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$$\lambda \int_{\Omega} e^{u_\lambda} \rightarrow 8k\pi \quad \text{as } \lambda \rightarrow 0,$$

then

$$\frac{1}{2} \nabla R(P_i) - \sum_{j=1, j \neq i}^k \nabla_x G(P_i, P_j) = \mathbf{0}$$

### Corollary (the case of 2 peaks)

Let us suppose to have a 2-peak solution to

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Then, we have that

$$\lambda \int_{\Omega} e^{u\lambda} \rightarrow 16\pi \quad \text{per } \lambda \rightarrow 0,$$

$$\begin{cases} \frac{1}{2} \nabla R(P_1) = \nabla_x G(P_1, P_2) \\ \frac{1}{2} \nabla R(P_2) = \nabla_x G(P_2, P_1). \end{cases}$$

## Theorem, Grossi and Takahashi (2010)

Let  $\Omega$  be convex and  $u_\lambda$  be a solution of

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with  $\|u_\lambda\| \rightarrow +\infty$  as  $\lambda \rightarrow 0$ . Then we have that,

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Hence, for  $\lambda$  small enough, all the solutions have just one peak.

### Lemma

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## Lemma

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$$\int_{\partial\Omega} (x - Q) \cdot \nu(x) \left( \frac{\partial G(x, P_1)}{\partial \nu_x} \right) \left( \frac{\partial G(x, P_2)}{\partial \nu_x} \right) ds_x$$

$$= (2 - N)G(P_1, P_2) + (Q - P_1) \cdot \nabla_x G(P_1, P_2) + (Q - P_2) \cdot \nabla_x G(P_2, P_1)$$

where  $Q \in \mathbb{R}^2$  and  $\nu(x)$  is the unit outer normal to  $x \in \partial\Omega$ .



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## Sketch of the proof

Let  $\Omega$  be convex and  $u_\lambda$  be a solution of

$$(P_\lambda) \quad \begin{cases} -\Delta u = \lambda e^u & \text{in } \Omega \subset \mathbf{R}^2, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

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## Sketch of the proof

- **Step1:** boundedness of  $\lambda \int e^u$
- **Step2:** nonexistence of  $k$ -peak solution with  $k \geq 2$  via the integral identity.

## Theorem

Let  $\Omega$  be convex and  $u_\epsilon$  be a solution of

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- **Location of the peaks.** It is (almost) the same of the two dimensional problem, (Bahri, Li e Rey (1995)).
- **Convexity of the Robin function.** In dimensions  $N \geq 3$  the convexity of the Robin function was proved by Cardaliaguet and Tahraoui (2002).

Thank you

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