HOMOGENIZATION OF DEGENERATE POROUS MEDIUM TYPE EQUATIONS IN ERGODIC ALGEBRAS

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ABSTRACT. In this talk we present some recent results on homogenization of nonlinear PDE's obtained in colaboration with Luigi Ambrosio and Jean Carlos Silva. We will review the concepts of algebra with mean value and ergodic algebra introduced by Zhikov and Krivenko (1986). We also introduce the Fourier-Stieltjes algebra in \mathbb{R}^n which we denote by $FS(\mathbb{R}^n)$. It is a subalgebra of the algebra of bounded uniformly continuous functions in \mathbb{R}^n , $BUC(\mathbb{R}^n)$, strictly containing the almost periodic functions, whose elements are invariant by translations and possess a mean-value. Thus, it is a so called algebra with mean value. Namely, $FS(\mathbb{R}^n)$ is the closure in $BUC(\mathbb{R}^n)$, with the sup norm, of the real valued functions which may be represented by a Fourier-Stieltjes integral of a complex valued measure with finite total variation. We prove that it is an ergodic algebra and that it shares many interesting properties with the almost periodic functions. We show how two-scale Young measures can be constructed in the general context of algebras with mean value. We apply the framework developed to study the homogenization problem for a degenerate porous medium type equation of the general form $u_t = \Delta f(x, \frac{x}{\varepsilon}, u)$ with f(x, y, u) monotone increasing for $u \ge 0$, f(x, y, 0) = 0. We analyse both the Cauchy problem and the initial boundary value problem with Dirichlet boundary condition.

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