Polynomial vector fields with algebraic trajectories

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abstract: Hilbert schemes have their counterpart in the theory of polynomial differential equations, namely the spaces of foliations. While a general surface of degree $d \geq 4$ in P^3 contains no line –in fact, only complete intersection curves are allowed– those that do contain some line, correspond to a subvariety of codimension $d - 3$ and degree $\binom{d+1}{4}(3d^4 + 6d^3 + 17d^2 + 22d + 24)/4!$ in a suitable P^N. In the same vein, while a general foliation (i.e., roughly a polynomial vector field) of degree $d \geq 2$ has no algebraic leaf, those that do have, say in P^2, an invariant line, correspond to a subvariety of a suitable P^N of degree $3\binom{d+3}{4}$. We show how to derive formulas like this for a wider class of invariant curves such as lines, conics, twisted cubics... (Joint with Viviana Ferrer Cuadrado).