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Dynamics of an excitable glow-discharge plasma under external forcing

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Glow discharge plasma in the excitable regime shows rich dynamical behavior under external forcing. By perturbing the plasma with a subthreshold sawtooth periodic signal, we obtained small subthreshold oscillations that showed resonance with the perturbation frequency. The resonance phenomenon can be useful to estimate characteristic of an excitable system. However, for suprathreshold perturbation, frequency entrainment was observed. In this case, the system showed harmonic frequency entrainment for the perturbation frequencies greater than the characteristic frequency of the system and the excitable behavior for the perturbation frequencies well below the characteristic frequency. The experiments were performed in a glow-discharge plasma where excitability was achieved at a suitable discharge voltage and gas pressure.

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I. INTRODUCTION

External periodic forcing of suitable amplitude and frequency on nonlinear systems that show self-oscillations leads to a wide variety of nonlinear phenomena such as frequency entrainments, period pulling, quasiperiodicity, and many other nonlinear phenomena [1–10]. Plasma is one such nonlinear system, which also shows those phenomena and has been studied extensively in the literature [2–10]. For suitable plasma parameters, plasma may also show another kind of nonlinear behavior called excitability and plasma in the excitable regime may be termed as excitable plasma [10–13]. Though the excitable systems show rich dynamical behavior under external forcing [14–23], the dynamics of excitable plasma has been studied only for few cases under external forcing (more particularly under stochastic forcing) [11–13]. In this paper, our aim is to explore the dynamics of a glow-discharge plasma in the excitable region under two types of periodic perturbations: subthreshold perturbation, i.e., the perturbation whose amplitude is not sufficient to excite the system to excited state, and suprathreshold perturbation, i.e., the perturbation whose amplitude is large enough to excite the system.

The basic characteristic of an excitable system is that it shows a fixed-point or coherent limit cycle oscillations depending on the value of the control parameter (CP) of the dynamics. The value of CP where the system changes from oscillatory to fixed-point behavior is called the threshold or bifurcation point. If the excitable system shows a fixed-point behavior for the value of CP that is below or above the threshold, the system will certainly show limit cycle oscillations for CP greater or lesser than the threshold value. The response of an excitable system (at the fixed-point state to an external perturbation applied on the CP) depends on the perturbation amplitude. When the perturbation is subthreshold, the system shows small oscillations around the fixed point [19]. These subthreshold or small oscillations may bear the characteristic of system dynamics [24–26]. However, when the perturbation is suprathreshold, the system returns to its fixed point deterministically; i.e., once the threshold is crossed, the system becomes almost independent of the perturbation and comes to its fixed-point state traversing one limit cycle [14–17]. In plasmas, excitability may be obtained through Hopf [10,12] or homoclinic bifurcation [11], depending on the plasma parameters.

Usually, the dynamics of periodically perturbed plasma that show self-oscillations (but not in excitable region) depends on both the amplitude and frequency of the external perturbation that may be represented by the "Arnold tongue" diagram. Though the dynamics of an excitable system depends on the frequency of the perturbation, it does not depend much on the perturbation amplitudes as long as the perturbation lifts the system to its excited state. This is because the variation in perturbation amplitude does not affect much the system dynamics at excited state [19], and hence the usual Arnold tongue diagram may not be obtained. So in the present experiments, the dynamics has been studied by varying frequency of the external perturbation.

The rest of the paper is organized as follows: we present a brief description of the experimental setup in Sec. II and autonomous system dynamics through which excitability was obtained in the glow-discharge plasma is discussed in Sec. III. Experimental results are presented and discussed in Sec. IV. Finally, we conclude the results in Sec. V.

II. EXPERIMENTAL SETUP

The experiments were carried out in a glow-discharge argon plasma produced by a dc discharge in a cylindrical hollow cathode electrode system with a typical plasma density

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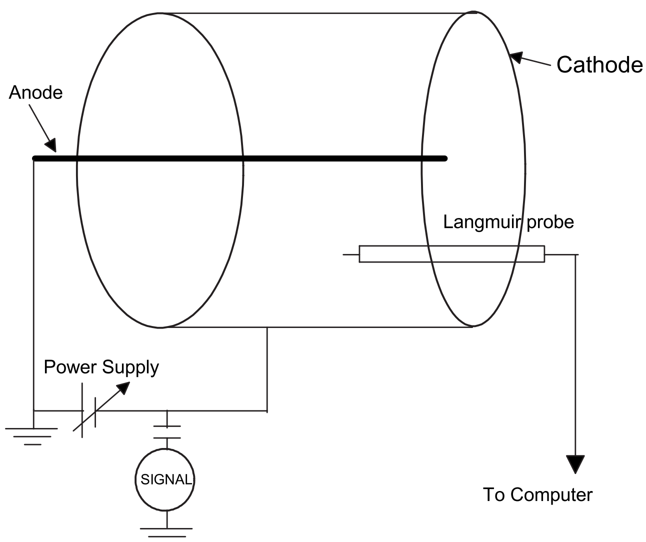

FIG. 1. Schematic diagram of the cylindrical electrode system of the glow-discharge plasma. The probe is placed at a distance $l \approx 12.5$ mm from the anode.

and temperature of $\approx 10^7$ cm^{-3} and 2–6 eV, respectively. The schematic diagram of the electrode system is shown in Fig. 1. The electrode assembly was housed inside a vacuum chamber, and the neutral pressure inside the chamber, whose range was between 0.001 to a few millibars, was controlled by a needle valve. The discharge voltage (DV) (0–999 V) was applied between the cathode and the anode keeping the anode grounded. As the anode was kept grounded, the applied voltage is basically negative. A signal generator (Fluke PM5138A), through which sawtooth signals were generated, was coupled with the dc power supply of the discharge plasma through a capacitor. The main diagnostics was the Langmuir probe, which was used to record the floating potential fluctuations. Detail of the experimental setup can be found in Ref. [11].

III. AUTONOMOUS DYNAMICS

By varying the neutral pressure, the discharge was obtained at different DVs. At higher pressure (i.e., order of 1 mbar), with the initiation of discharge, a bright glow or spot, which was unstable in nature, was observed to form on the anode, and this unstable glow was the source of the chaotic fluctuations in the plasma floating potential. An estimate of the frequency of these instabilities can be obtained from the ion transit time in the plasma [8,27], $\tau(d)=\frac{d}{V_{th,i}}=d/\sqrt{\frac{k_bT_i}{m}}$, where d is the electrode distance. The estimated ion transit frequency $(1/\tau)$ for our experimental system is on the order of a few kilohertz, which agrees well with the frequency of the fluctuations. As the frequencies of these oscillations were within the ion plasma frequency and the presence of anode glow and relaxation oscillations have been attributed to the presence of double layers, these fluctuations could be related to ion acoustic instabilities and anode glow related double layer. Detail of the characterization of the fluctuations has been presented in Ref. [27].

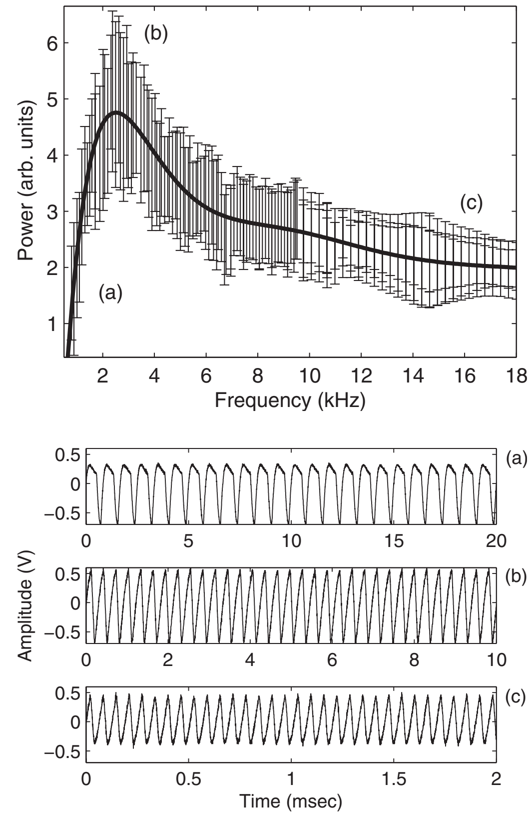

FIG. 2. Upper panel: resonance curve of the subthreshold oscillations with frequency of the applied sawtooth signals. It peaks at system internal frequency around 2.7 kHz. Lower panel: time series at the points (a) before resonance, (b) around resonance, and (c) after resonance, and these points have also been shown in power spectra (upper panel).

These fluctuations changed their behavior with change in pressure as well as DV. At the initial stage of discharge, system showed chaotic oscillations, which changed to relaxation type oscillations with increase in DV. Increasing in the DV led to an increase in the amplitude and the time period of the oscillations and became regular (period 1) at certain point of DV. Further increase in the DV modified the oscillation profile and resulted in typical relaxation oscillations, which changed to excitable fixed-point behavior at a certain point through homoclinic bifurcation. The DV at which these oscillations cease may be termed as bifurcation point (V_H), which depends also on the neutral pressure. The higher the neutral pressure is, the higher the V_H is. The time period (T) of these relaxation oscillations increased with increasing DV near the bifurcation point and became infinite beyond the V_H, resulting in vanishing of the limit cycles. For larger values of DV, the autonomous dynamics exhibited a steady-state fixed-point behavior. The detail of the autonomous dynamics through which system attains excitability has been presented in Ref. [11].

In these experiments, DV, which was the control parameter of the excitable dynamics, was so chosen that the system exhibited fixed-point behavior and, subsequently, DV was perturbed by subthreshold and suprathreshold external sawtooth periodic signals.

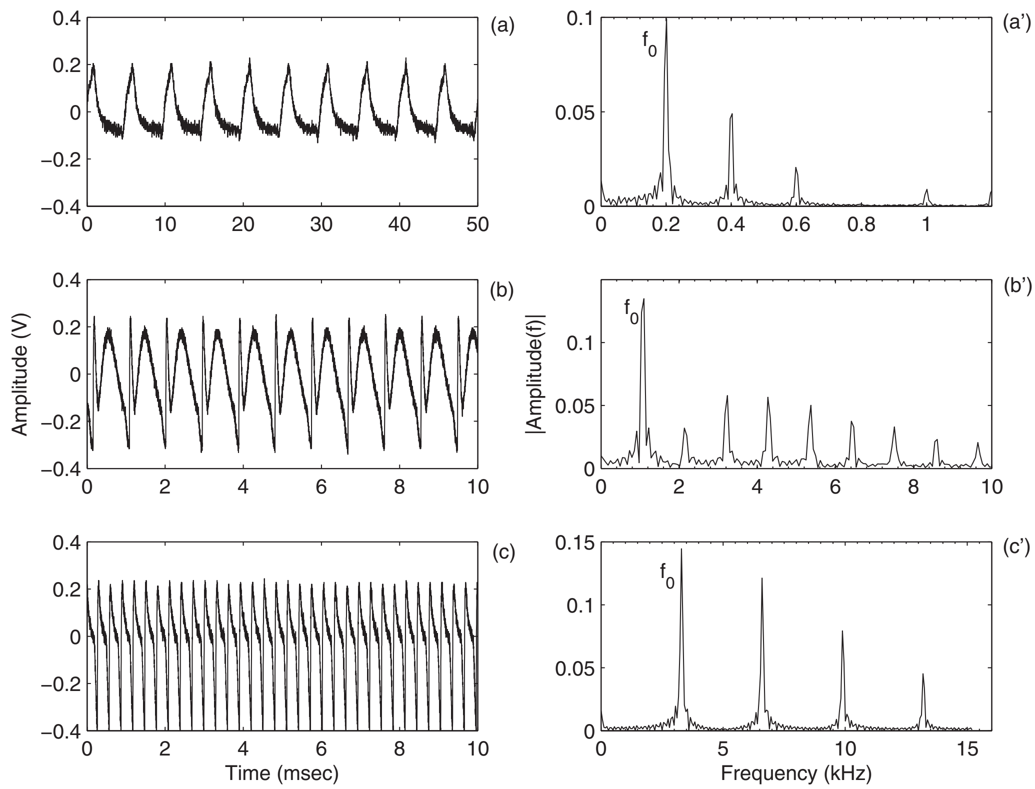

FIG. 3. Left panel: time series of plasma floating potential for periodic sawtooth perturbation: (a) at frequency 0.2 kHz; (b) at 1.072; and (c) at 3.56 kHz and their power spectra at (a′), (b′), and (c′), respectively. (a) shows that system does not respond at frequency of 0.2 kHz; (b) shows the system and its excitable behavior at 1.072 kHz; and (c) shows 1:1 entrained state at 3.56 kHz. 1:1 entrained state is also clear from the power spectrum (c′) also where internal frequency of the system coincides with external perturbation.

IV. RESULTS

The experimental results of the dynamical behavior of the excitable plasma under subthreshold and suprathreshold have been presented in Secs. IV A and IV B, respectively.

A. Effect of subthreshold signal

For the study of the effect of a subthreshold signal, the discharge voltage ($|V_0| \approx 429$ V) was chosen such that $|V_0| \gg |V_H| \approx 314$ V and therefore the system dynamics, by virtue of an underlying homoclinic bifurcation, exhibits fixed-point behavior, and the pressure was equal to 0.46 mbar during these experiments. To avoid parametric drift V_0 was chosen away from V_H. The applied DV is negative as anode was grounded during experiment. The discharge voltage was thereafter perturbed, $V=V_0+S(t)$, where a subthreshold sawtooth periodic signal $[S(t)]$ of peak to peak amplitude of ~ 10 V was so chosen that the voltage $V=V_0+S(t)<V_H$; i.e., the applied perturbation does not cause the system to cross over to the oscillatory regime. The subthreshold periodic sawtooth signal of variable frequency was generated by using a Fluke PM5138A function generator. In this experiment, the frequency of the applied signal was varied from a few hertz to 20 kHz. When the frequency of the applied signal was around a few hundred hertz, system showed subthreshold oscillations. The amplitude of these oscillations increased with the increase in the perturbation frequency up to a few kilohertz and after that the amplitude decreased again.

Figure 2 (upper panel) shows the power of the subthreshold oscillations as a function of the frequency of the applied signals. Power has been estimated using the fast Fourier transform. It shows that the power of the subthreshold oscillations was low initially and increased with increase in frequency. Power was maximum around 2.7 kHz. On further increasing of frequency power decreased; hence, the system showed resonance at 2.7 kHz. Figures 2(a)–2(c) (lower panel) are the time series when the plasma was perturbed by sawtooth signals of frequencies 1.2, 3, and 16 kHz, respectively, which show that the amplitude of the time series at resonance point (2.7 kHz) is largest. As the frequency of the internal plasma oscillations was also on the order of a few kilohertz [27,28], the resonance may be used to determine the frequency of internal oscillations of plasma system.

Contrary to our observations, in some earlier works [24,25], it is observed that these subthreshold oscillations change to chaotic oscillations through some route on changing the frequency of the perturbations; this may be due to the application of forcing near the threshold. As these oscillations were not observed far away from the threshold, these oscillations are the feature of excitable dynamics [19].

B. Effect of suprathreshold signal

For the experiments on the effect of a suprathreshold forcing, the DV ($|V_0| \approx 334$ V) was set such that the floating

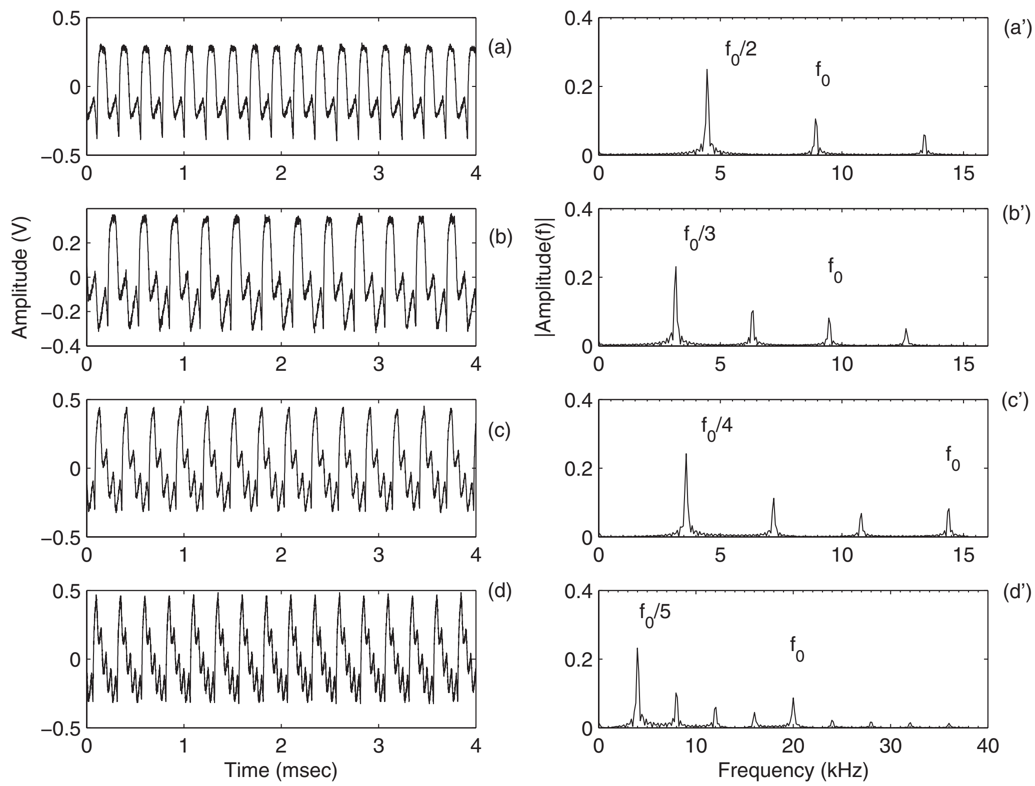

FIG. 4. Time series of plasma floating potential for periodic sawtooth perturbation: (a) at frequency of 8.93 kHz; (b) at 9.48; (c) at 14.38 kHz; and (d) at 20.0 kHz. (a) shows 1:2 entrained; (b) shows 1:3 entrained state; (c) shows 1:4 entrained state; and (d) shows 1:5 entrained state.

potential fluctuations exhibit fixed-point behavior, and the pressure was kept 0.46 mbar as that of subthreshold forcing experiments. In order to minimize the effect of parameter drift, a set point (V_0) quite far from the homoclinic bifurcation ($|V_H| \approx 310$ V) was chosen. Subsequently, a suprathreshold sawtooth signal of fixed amplitude (peak to peak $\sim$25 V) but of variable frequency was superimposed on the DV so that the perturbation always crosses the threshold, i.e., $V = V_0 + S(t) > V_H$. The frequency of the applied signals was varied from a few hertz to 20 kHz.

At low frequency, i.e., less than 1 kHz, though the amplitude of the applied signal is sufficient to cross the threshold, the system did not show any excitable dynamics. At these frequencies, we obtained only the plasma distorted sawtooth signals as shown in Fig. 3(a) for perturbation frequency of 0.2 kHz. We obtained perturbation invoked limit cycle oscillations at frequency of 1.072 kHz as shown in Fig. 3(b). In the figure the oscillations with smaller time period are those oscillations as their time period is on the order of system's characteristic time scale (inverse of internal frequency), and oscillations with larger time period are those of the applied signal. In this stage, system dynamics (smaller time period) may not be taken as phase locked with applied signal as they have appeared due to the crossing of threshold. Figure 3(b) indicates that an excitable system also has a threshold frequency that has to be crossed to invoke excitable dynamics.

Figures 3(c) and 3(c′) show the output time series and corresponding power spectrum at the perturbation frequency equal to 3.56 kHz. They clearly show that the system response is phased locked with the external periodic signal with rotation number 1:1. In case of phase-locked states, the rotation number (w) is defined as $W = f/f_0$, where f and f_0 are the frequencies of external perturbation and system frequency, respectively [9]. So when the oscillation frequency of the system was close to perturbation frequency ($f \approx f_0$), 1:1 phase locking or entrainment phenomenon was obtained. Though perturbations invoked limit cycle oscillations were obtained both at frequencies of 1.072 (Fig. 3) and 3.56 kHz [Fig. 3(c)], the first case [Fig. 3(b)] cannot be taken as frequency entrainment phenomenon, as the phase is not locked with the external perturbation, which may be due to the large mismatch between the time scales of internal plasma dynamics and external perturbation. In second case [Fig. 3(b)], external perturbation lifts the system only at its excited state and invokes limit cycle oscillations that are typical of excitable dynamics; whereas at 3.56 kHz [Fig. 3(c)], external perturbations lift the system at excited state as well as lock to the phase of internal oscillations. So the behavior of an excitable plasma under an external suprathreshold perturbation depends both on its excitable and internal dynamics.

When the perturbation frequency was increased to 8.93 kHz, system showed 1:2 harmonic frequency entrainment as shown in Fig. 4(a) and corresponding power spectrum shown in Fig. 4(a′). In Fig. 4(a′), f_0 represents the frequency of external perturbation and $f_0/2$ is that of the system oscillation at excited state. At perturbation frequencies of 9.48, 14.38, and 20.0 kHz, we obtained 1:3, 1:4, and 1:5 entrainments or phase locked states, respectively. These entrainment

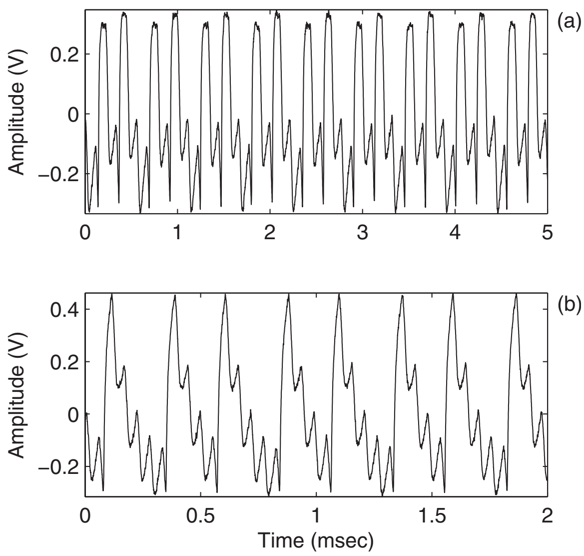

FIG. 5. Typical time series of plasma floating potential which show (a) switching patterns between 1:2 and 1:3 at frequency of 9.05 kHz and (b) switching patterns between 1:4 and 1:5 at frequency of 18.29 kHz.

phenomena are also clear from the times series shown in Figs. 4(b)–4(d) and from their power spectra shown in Figs. 4(b′), 4(c′), and 4(d′), respectively.

Between any two phase-locked states, the system responded in such a way that it showed switching patterns. Figures 5(a) and 5(b) show the typical switching patterns between 1:2 and 1:3; and 1:4 and 1:5, respectively. We did not get any switching patterns between 1:3 and 1:4; this may be due to the coarse tuning of the DV.

The above results show that the low-frequency perturbations (i.e., well below its intrinsic frequency) lift the system only to its excited state, and hence system shows limit cycle oscillations that are the characteristic of an excitable system, but such a perturbation does not produce any entrained state; however, plasmas that exhibit nonlinear self-oscillations or in other words van der Pol–type behavior generally show subharmonic entrainments (i.e., 2:1, 3:1, etc., frequency entrainments) under low frequency [3,9]. Another important difference between the behavior of an excitable plasma and the plasma that show nonlinear self-oscillations, i.e., van der Pol type oscillations, is that first one does not show the Arnold tongue behavior since the perturbation amplitude does not play much role in the dynamics of excitable systems [3,9].

As the resolution of the power supply used in these experiments was 1 V, we may have missed some dynamic features. The absence of the switching patterns between 1:3 and 1:4 may be due to this. There may also be some problem due to the coupling of external signal through a capacitor as very low and high frequencies are blocked in the capacitor. Better power supply and coupling may produce other features of the dynamics.

The simplest equations compatible with the dynamical scenario of the excitable system are the FitzHugh-Nagumo model. Appearance of stochastic or coherence resonance in excitable plasma has been modeled using these simple models in Ref. [13]. However, the appearance of the subthreshold oscillations and frequency entrainments are not fully understood for excitable plasma, and we may need more general model [1] that accommodates more numbers of parameters and consistent with basic plasma processes. One probably has to carry out a detailed simulation to investigate these phenomena.

V. CONCLUSION

In this paper we have investigated the dynamical behavior of an excitable glow-discharge plasma under the subthreshold and suprathreshold periodic forcing. The excitable system shows subthreshold oscillations under subthreshold forcing, wherein these oscillations are typical of the system dynamics, and there is a resonance between the external subthreshold perturbations and the internal frequency of the system. This resonance behavior may be useful to characterize an excitable plasma and may also be applied in other excitable systems.

For the suprathreshold perturbation, the system shows an excitable or mode locking behavior depending on the perturbation frequency. The most simple frequency entrainment (1:1) phenomenon was obtained when the driver frequency was chosen close to resonance frequency that was obtained during the subthreshold forcing (Sec. IV A), and for the perturbation frequency lesser than the resonance frequency the system showed only excitable dynamics; however, for higher than the resonance frequency the system showed harmonic entrainments. As the dynamics of an excitable system at excited state is almost independent of the amplitude of external perturbation, we did not get the Arnold tongue behavior in the excitable region of glow-discharge plasma.

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