META-MODELING FOR ROBUST DESIGN AND MULTI-LEVEL OPTIMIZATION

R. Duvigneau and Praveen C.
Opale project-team, INRIA Sophia Antipolis (France)
Regis.Duvigneau@sophia.inria.fr Praveen.Chandrashekarappa@sophia.inria.fr

ABSTRACT

In this paper, we propose to study meta-modeling techniques in order to develop a two-level modeling procedure for aerodynamic design. Two typical problems are successively examined: the acceleration of a genetic algorithm by using an inexact pre-evaluation approach and the estimation of statistics for robust design under uncertain operating conditions. Radial basis functions are employed to approximate the aerodynamic coefficients in both cases. The proposed approach is applied to optimize the wing shape of a business aircraft.

INTRODUCTION

Optimum shape design in aerodynamics has been an active research topic for the last years. Deterministic (gradient-based) and semi-stochastic (evolutionary) methods provide solutions to perform both global and accurate search. However, the computational cost related to the flow analysis using Computational Fluid Dynamics (CFD) codes is still an obstacle to extend the current results to complex industrial problems.

The present study aims at developing a two-level modeling strategy to address this difficulty. We propose to include in the design procedure a low-cost modeling technique beside the expensive CFD solver, which allows to avoid some expensive evaluations without degrading the accuracy of the result.

Particularly, Meta-Models (MMs), i.e. models of models, are under consideration here as methods to interpolate data from prior computations, thanks to their capability to predict non-linear behaviors. However, our strategy can possibly use other simplified modeling, such as low-fidelity physics or coarse grid computations.

The proposed two-level modeling strategy is applied to accelerate a genetic algorithm and estimate statistics for a robust design problem.

META-MODELS

Principles

MMs are surrogate models whose evaluations are far less expensive than the original model (PDEs solving for instance). Then, they can be used for a negligible cost to remplace some evaluations of the original model or to provide additional information.

MMs are constructed according to available data that are stored in a database. This database can be generated separately or compiled during optimization. The MMs and the database used for the construction can be either global or local. In the former case, the database should cover the whole design space, whereas in the latter case the database only contains data located in a prescribed region of the design space. MMs mostly used for data fitting are:

- polynomial fitting (least-squares approximation) ;
- artificial neural networks (multi-layer perceptrons) [13];
- radial basis functions [11] ;
- Kriging methods (Gaussian process models) [15].

Although the last three options are well suited to highly non-linear behaviors, such as those encountered in aerodynamics, the present study only uses Radial Basis Functions (RBFs).

MMs can be employed in numerical optimization according to one of the two following strategies:

1. Generate *a priori* a global database, construct one MM and then solve the optimization problem using only the MM evaluations. The original model is then used *a posteriori* to evaluate the fitness of the optimal design found.
2. Solve the optimization problem using both the original model and several MMs, which are constructed using local databases that compile some previous evaluations, in an interactive multi-level evaluation strategy.

The first option is usually used iteratively by including in the database new data computed in the vicinity of the optimum design found. However, this option relies on a weak coupling between the models and can lead to local optima. If the whole design space is not filled enough by the data contained in the database, the MM can lead the optimizer to non-optimal design. On the contrary, the second strategy relies on a two-level modeling approach, that allows a progressive database filling in most promising regions. This strong coupling between the models yields a more robust algorithm.

This two-level modeling strategy is demonstrated in the present study for two different applications in aerodynamic design: the acceleration of a genetic algorithm and the estimation of uncertainty in robust design.

Radial basis functions

RBFs are non-polynomial interpolation methods for scattered data [11]. They have been found to be very accurate for highly non-linear data in high dimension [3]. RBFs seek an approximation of the function $\mathcal{J}(\mathbf{x})$, $\mathbf{x} \in \Re^n$ of the form:

$$\tilde{\mathcal{J}}(\mathbf{x}) = \sum_{j=1}^{N_c} \omega_j \ \phi_j(\mathbf{x}), \qquad (1)$$

where:

$$\phi_j(\mathbf{x}) = \Phi(\|\mathbf{x} - \mathbf{x}_j\|). \qquad (2)$$

$(\mathbf{x}_j)_{j=1,\dots,N_c}$ are called RBFs centers. Several radial functions Φ can be considered. For the present study a Gaussian function is employed:

$$\Phi(r) = e^{-\frac{r^2}{s^2}}, \tag{3}$$

where s is a parameter called attenuation factor. Therefore, the evaluation of the RBFs using (1-3) has a negligible computational cost.

The training of the RBFs consists in determining the weights $(\omega_j)_{j=1,\dots,N_c}$ to fit the data. Suppose that the function value is known for a set of N_c points that correspond to the RBF centers $(\mathbf{x}_i)_{i=1,\dots,N_c}$. Then, the weights $(\omega_j)_{j=1,\dots,N_c}$ are determined from the interpolation conditions:

$$\mathcal{J}(\mathbf{x_i}) = \sum_{j=1}^{N_c} \omega_j \ \phi_j(\mathbf{x}_i) \quad i = 1, \dots, N_c. \tag{4}$$

Thus, $(\omega_j)_{j=1,\dots,N_c}$ is the solution of the following linear system:

$$\begin{pmatrix} \phi_1(\mathbf{x}_1) & \dots & \phi_{N_c}(\mathbf{x}_1) \\ \phi_1(\mathbf{x}_2) & \dots & \phi_{N_c}(\mathbf{x}_2) \\ \dots & \dots & \dots \\ \phi_1(\mathbf{x}_{N_c}) & \dots & \phi_{N_c}(\mathbf{x}_{N_c}) \end{pmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \dots \\ \omega_{N_c} \end{Bmatrix} = \begin{Bmatrix} \mathcal{J}(\mathbf{x}_1) \\ \mathcal{J}(\mathbf{x}_2) \\ \dots \\ \mathcal{J}(\mathbf{x}_{N_c}) \end{Bmatrix}. \tag{5}$$

The matrix of the system is obviously symmetric. It is also positive-definite if the RBF centers $(\mathbf{x}_j)_{j=1,\dots,N_c}$ are distinct. One can notice that it is a full matrix. However, its dimension N_c is usually moderate in practice. Then, the computational cost of its inversion is far lower than that of PDEs solving. The attenuation factor s can be determined experimentally by the user. The following empirical formula is proposed in [10]:

$$s = d_{max}(nN_c)^{-\frac{1}{n}}, \tag{6}$$

where n is the dimension of the problem and d_{max} the maximum distance between RBF centers. However, the choice of the attenuation factor can be tedious for some applications. Then, we propose to optimize it by minimizing an error estimation functional. According to [14], we employ the *leave-one-out* technique: one point of the data set is ignored during the training. This point is then used to estimate the fitting error. By considering successively all the points of the data set, one can estimate a global error for a given attenuation factor. Thus, a Particle Swarm Optimization (PSO) algorithm is used to determine the attenuation factor that minimises this error.

Multi-level evolutionary optimization

Methodology

Genetic Algorithms (GAs) are optimization methods that mimic the Darwinian evolution process. They are able to find global optima for multimodal or even discontinuous cost functions. However, a large number of cost function evaluations is required to converge to the optimum design.

Consider a typical parametric shape optimization problem:

$$\begin{aligned} &\text{Minimize } \mathcal{J}(\mathbf{x}) \quad \mathbf{x} \in \Re^n, \\ &\text{Subject to } \mathcal{C}(\mathbf{x}) \leq 0, \end{aligned} \tag{7}$$

A classical GA to solve such a problem is organized as:

1. Evaluate the fitness of all individuals in the population ;
2. Apply the selection operator to eliminate non-promising individuals ;
3. Apply the crossover operator to generate offsprings from the selected individuals ;
4. Apply the mutation operator to modify randomly the offsprings.

Gianakoglou [6, 7] has proposed a two-level evaluation strategy, called Inexact Pre-Evaluation (IPE), to reduce the computational time related to GAs. It relies on the observation that numerous cost function evaluations are useless, since numerous individuals do not survive to the selection operator. Then, it is not necessary to determine their fitness accurately. The strategy proposed by Gianakoglou consists in using MMs to pre-evaluate the fitness of the individuals in the population. Then, only a small portion of the population, that corresponds to the most promising individuals, are accurately evaluated using the original and expensive model.

The database is generated progressively by including at each generation the data provided by the evaluations using the original model. Using such a strategy, the database is progressively enriched in the promising regions of the design space and the accuracy of the MMs is progressively improved.

In the next section this strategy is applied to the optimization of a wing shape in transonic regime.

Test-case

The test-case considered here corresponds to the optimization of the wing shape of a business aircraft (courtesy of Piaggio Aero Ind.). The test-case is described in depth in [1]. The free-stream Mach number is $M_\infty = 0.83$ and the incidence $\alpha = 2°$. Initially, the wing section is supposed to correspond to the NACA 0012 airfoil.

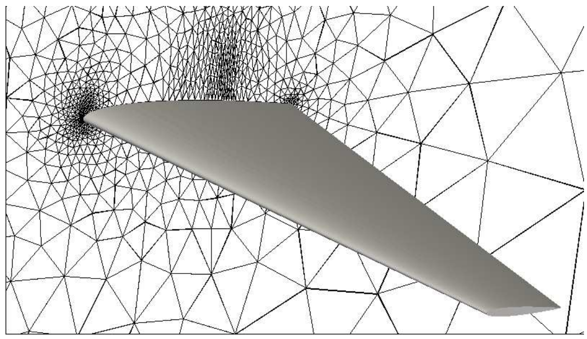

Figure 1: Initial wing shape and mesh in the symmetry plane.

The goal of the optimization is to reduce the drag coefficient C_D subject to the constraint that the lift coefficient C_L should not decrease more than 0.1%. The constraint is taken into account using a penalization approach. Then, the resulting cost function is :

$$\mathcal{J}_{OPT} = \frac{C_D}{C_{D0}} + 10^4 \ \max(0, 0.999 - \frac{C_L}{C_{L0}}). \tag{8}$$

C_{D0} and C_{L0} are respectively the drag and lift coefficients corresponding to the initial shape (NACA 0012 section). The aerodynamic coefficients are computed by simulating three-dimensional inviscid compressible flows governed by the Euler equations. An unstructured mesh, composed of 31124 nodes, is generated around the wing, including a refined area in the vicinity of the shock (see figure (1)).

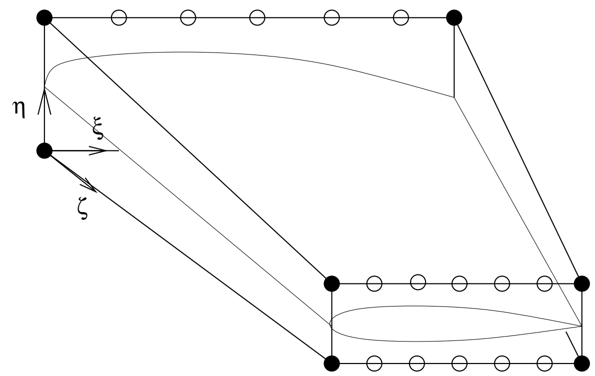

Figure 2: FFD lattice around the wing: moving control points (filled markers) and frozen control points (empty markers).

The shape parameterization is ensured by the Free-Form Deformation (FFD) approach [1, 4]. A FFD lattice is built around the wing with ξ, η and ζ in the chordwise, spanwise and thickness directions respectively. The lattice is chosen in order to fit the planform of the wing (see figure (2)). Then, the leading and trailing edges are kept fixed during the optimization by freezing the control points that correspond to $i = 0$ and $i = n_i$. Moreover, control points are only moved vertically. Three parameterization levels are under consideration in the next sections. The coarsest one corresponds to $n_i = 3$, $n_j = 1$ and $n_k = 1$. Therefore, $(4-2)\times 2\times 2 = 8$ degrees of freedom are taken into account in the optimization. The medium parameterization corresponds to $n_i = 6$, $n_j = 1$ and $n_k = 1$ and counts $(7-2)\times 2\times 2 = 20$ degrees of freedom. Finally, the finest parameterization corresponds to $n_i = 9$, $n_j = 1$ and $n_k = 1$ and counts $(10-2)\times 2\times 2 = 32$ degrees of freedom.

A real coding GA is used for this study. The selection is ensured by a roulette wheel operator, whereas a two-point crossover and a non-uniform mutation are used [9].

Results

The two-level modeling technique is tested on the shape optimization problem presented above, using the coarsest parameterization (8 d.o.f.). The population used by the GA counts 60 individuals. The evolution is simulated during 200 generations. The fitness is evaluated by CFD simulations during the first 10 generations, yielding an initial database. Then, three cases are studied:

- 100 % of evaluations use CFD simulations ;
- 30 % of evaluations use CFD simulations and 70 % use RBFs predictions ;
- 20 % of evaluations use CFD simulations and 80 % use RBFs predictions ;

RBFs are trained for each individual, using a local database which includes 16 points, with an optimized attenuation factor.

The comparison of the results obtained is shown in figure 3, which depicts the cost function value with respect to the number of CFD evaluations, and in table 1. As can be seen, the use of RBFs for 70 % of the population yields results similar to those obtained with a classical GA. The number of evaluations using the CFD code is reduced by a factor close to three. However, if one reduces the ratio of evaluations using the CFD solver to 20 %, one observes a degradation of the results. In that case, the progressive filling of the database is too low in order the RBFs to provide suitable information to the optimizer.

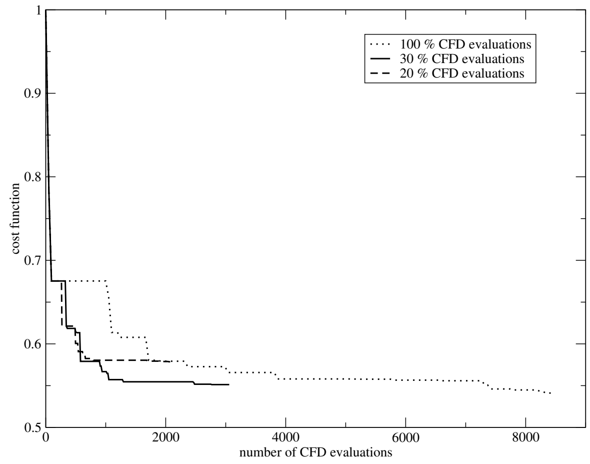

Figure 3: Evolution of the cost function w.r.t. the number of CFD evaluations.

Case	Cost funct.	Numb. of eval.
100 % CFD	0.5411	8431
30 % CFD	0.5512	3058
20 % CFD	0.5785	2074

Table 1: Results for the two-level evolutionary algorithm.

A comparison of the shapes obtained with the classical GA and with the GA using 30 % of CFD evaluations is provided by figure 4. As can be seen, similar shapes are obtained, although the computational cost is significantly reduced.

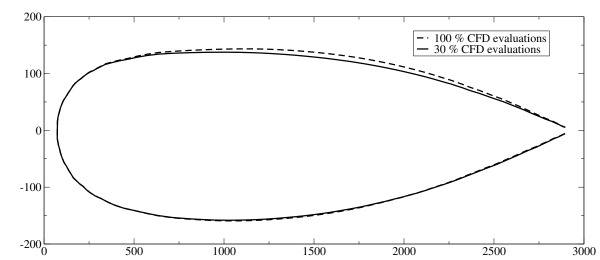

Figure 4: Comparison of the shapes at the root section : with and without inexact pre-evaluation procedure.

ROBUST DESIGN

Introduction to robust design

Application of classical design methods to solve the problem (7) can lead to unexpected performance losses in practice and unacceptable results. Indeed, the prescribed optimized design is subject to inherent geometrical variations due to manufacturing tolerances. Moreover, operating conditions, such as Mach number, angle of attack, etc, are subject to variability and random fluctuations. Therefore, the fitness of the optimal design predicted by CFD is usually not obtained in practice, due to geometrical uncertainty and operating condition uncertainty.

To overcome this difficulty, robust and reliability-based design methods are developed, that assess the effects of uncertainty on the design performance during the optimization procedure. The objective of *robust design* methods is to minimize the performance loss due to everyday fluctuations, whereas the aim of *reliability-based design* methods is to manage the consequences of extreme events. The present work is focused on the development of numerical methods for robust design in the framework of realistic aerodynamic problems. Particularly, the optimization with uncertain operating conditions is addressed.

Methodology

In a statistical approach, one considers the fluctuating operating conditions (Mach number, angle of attack, etc)) $\mathbf{a} = (a^m)_{m=1,...,M}$ as samples of random variables $\mathbf{A} = (A^m)_{m=1,...,M}$, whose statistical characteristics are known (mean $\boldsymbol{\mu}_\mathbf{A} = (\mu_A^m)_{m=1,...,M}$, variance $\boldsymbol{\sigma}^2_\mathbf{A} = ({\sigma_A^2}^m)_{m=1,...,M}$, etc). One also supposes for the sake of simplicity that the random variables $(A^m)_{m=1,...,M}$ are independent. The statistical characteristics of operating conditions can be determined by experimental measurements or engineering experience. Gaussian Probability Density Functions (PDFs) or truncated Gaussian PDFs are often used in practice (see [17] for instance).

The main consequence of this assumption is that the cost function of the system is also a random variable J. According to the Von Neumann-Morgenstern decision theory [2], the best choice is then to select the design which leads to the best expected fitness. This is known as the Maximum Expected Value (MEV) criterion. In practice, it consists in minimizing the statistical mean μ_J of the cost function. However, this strategy does not address the variability of the fitness. For engineering problems, one also would like to select a design for which the fitness is not subject to large variations as operating conditions fluctuate. Then, a second criterion is often joined to the MEV criterion, that relies on the minimization of the fitness variance σ_J^2:

$$\text{Minimize} \begin{cases} \mu_J = \displaystyle\int_{\Omega(\mathbf{A})} \mathcal{J}(\mathbf{x}, \mathbf{a})\, \rho_\mathbf{A}(\mathbf{a})\, d\mathbf{a} \\ \sigma_J^2 = \displaystyle\int_{\Omega(\mathbf{A})} (\mathcal{J}(\mathbf{x}, \mathbf{a})) - \mu_J)^2\, \rho_\mathbf{A}(\mathbf{a})\, d\mathbf{a} \end{cases}$$
$$\text{Subject to } P[\mathcal{C}(\mathbf{x}, \mathbf{A}) \leq 0] \geq p \quad \mathbf{x} \in \Re^n. \tag{9}$$

$\Omega(\mathbf{A})$ is the range of the random variable $\mathbf{A}$ and $\rho_\mathbf{A}$ is the PDF of $\mathbf{A}$. One can notice that, contrary to the classical problem (7), the constraint is now expressed with a probabilistic formulation: the probability P that the constraint $\mathcal{C}(\mathbf{x}, \mathbf{A})) \leq 0$ is verified should be larger than a prescribed value p. This approach aims at determining a trade-off between the expected fitness and the expected fitness variation as operating conditions randomly fluctuate. It is a significant improvement over previous methods, such as multi-point optimization. The robust design problem is now considered within a rigorous statistical framework. This allows to take into account the random fluctuations of the fitness in the optimization problem, but also to care about the frequency of occurence of the events, thanks to PDFs. Then, the most probable events have a larger influence in the decision than extreme and unlikely events. Although this approach is satisfactory from theorical and practical viewpoints, its application is not straigthforward. Particularly, the estimation of the mean and variance can be tedious for complex CFD applications. The random variable J is an output of a complex numerical simulation tool, such as a CFD solver, whereas $\mathbf{A}$ is an input parameter. Therefore, the main issue is the estimation of output uncertainty according to the knowledge of input uncertainty, i.e. the propagation of uncertainty through the CFD solver [12, 16, 17].

A classical approach to estimate statistics of a random variable is to use Monte-Carlo methods. A sample of operating conditions $(\mathbf{a}_i)_{i=1,...,N}$ of size N is generated according to the PDF $\rho_\mathbf{A}$. Then, unbiased estimators of the mean and variance are:

$$\mathcal{M}_J = \frac{1}{N} \sum_{i=1}^{N} \mathcal{J}(\mathbf{x}, \mathbf{a}_i), \tag{10}$$

$$\mathcal{S}_J^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\mathcal{J}(\mathbf{x}, \mathbf{a}_i) - \mathcal{M}_J)^2. \tag{11}$$

However, it is well known that this stochastic approach requires large samples to provide an accurate estimation of the variance. For CFD applications, a direct Monte-Carlo method is not conceivable presently.

To overcome this difficulty, we propose according to [8] to employ a two-level evaluation strategy. For each design $\mathbf{x}$, a small number N_c of cost function evaluations is performed using the original model with variable operating conditions. These evaluations $\mathcal{J}(\mathbf{x}, \mathbf{a}_i)$ $i = 1, \ldots, N_c$ are used as database to construct a MM $\tilde{\mathcal{J}}(\mathbf{x}, \mathbf{A})$ that describes the effect of the random variable $\mathbf{A}$ on the cost function for the considered design. Then, the MM is used in a Monte-Carlo approach to estimate the mean and the variance of the cost function, according to (10-11). Since all evaluations during the Monte-Carlo estimation are using the MM, the computational cost is negligible. In the next section, this approach is demonstrated for robust wing design.

Test-case

The optimization of the wing shape presented above is revisited in the context of robust design, with the finest parameterization (32 d.o.f.). We suppose that the free-stream Mach number is subject to random fluctuation. For simplicity, we assume that its PDF is Gaussian with a given mean μ_M and variance σ_M. The mean Mach number corresponds to the nominal Mach number $\mu_M = 0.83$ and its standard deviation is $\sigma_M = 0.0166$. Figure 5 depicts the Mach number distribution.

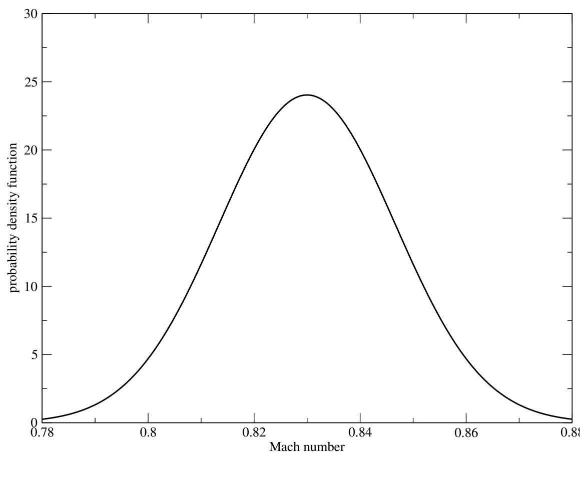

Figure 5: Probability density function of the free-stream Mach number.

The behavior of the initial design (NACA 0012 section) and the optimum design found solving the classical problem (7) is first analyzed for a Mach number that varies around its nominal value. Twenty-one flow analyses are performed in the interval $[\mu_M - 3\sigma_M, \mu_M + 3\sigma_M]$. Results are depicted in figure 6. As expected, the optimum design is fully adapted to the nominal Mach number: the drag coefficient is maintained at a low value until the nominal Mach number and then increases abruptly.

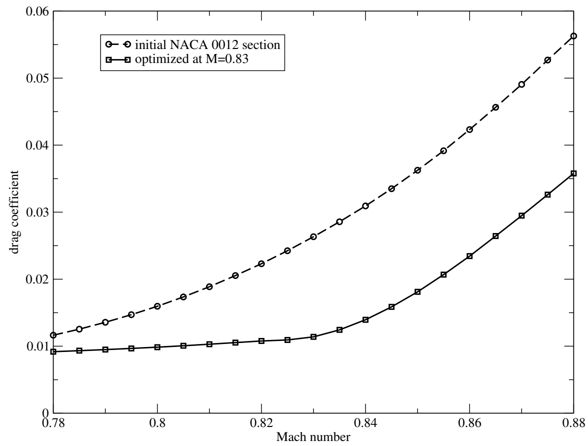

Figure 6: Drag variation for fluctuating Mach number: initial design and optimum design at nominal Mach number (0.83).

This observation motivates the robust design of the wing subject to uncertain Mach number. Then, the classical problem (7) is replaced by the robust formulation (9). Our purpose is now to reduce the mean drag as well as its variance. The lift constraint is now: the probability for the lift to be higher than the prescribed value should be higher than $p = 0.95$.

Uncertainty propagation

We study in this section the uncertainty propagation approach proposed above, that uses a two-level evaluation strategy. We try to determine the PDF of the drag coefficient for the optimum design found previously, as well as its mean and its variance. First, a reference result is obtained by using the previous twenty-one analyses to construct a very fine model (RBFs) and propagate the uncertainty by the Monte-Carlo approach. This reference result is then compared to the use of three possible models that rely on $N_c = 5$ points in the database: linear fitting, quadratic fitting and RBFs. In this case, RBFs use the empirical value for the attenuation factor (6). The five points are uniformly distributed in the interval $[\mu_M - 3\sigma_M, \mu_M + 3\sigma_M]$. Figure 7 shows the data fitting for the linear and quadratic least-squares approximations. The resulting PDF are compared to the reference result in figures 8 and 9.

The linear fitting has obviously a poor accuracy and the resulting PDF is Gaussian. This is far from the reference result, for which the PDF has a more complex shape and is characterized by a peak at low drag values. The quadratic fitting is closer to the CFD calculations at high Mach numbers. Then, the tail of the PDF is quite well reproduced. However, the peak description is not satisfactory.

Figures 10 and 11 depict the results obtained with RBFs. Obviously, the capability of RBFs to fit the data is better than that of linear or quadratic functions. The resulting PDF is similar to the reference PDF, except for the peak intensity. This is due to the discrepancy that can be observed be-

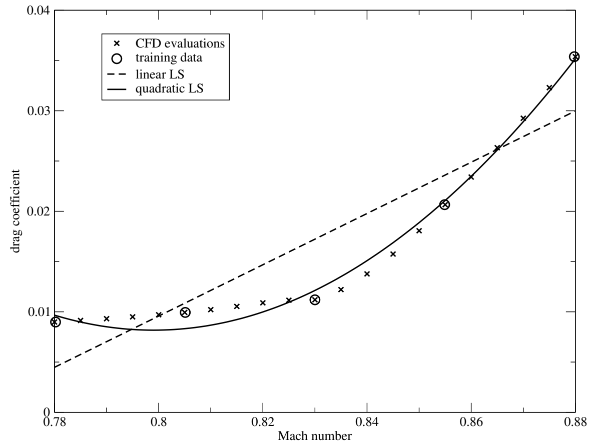

Figure 7: Five-point fitting using linear and quadratic least-squares approximation.

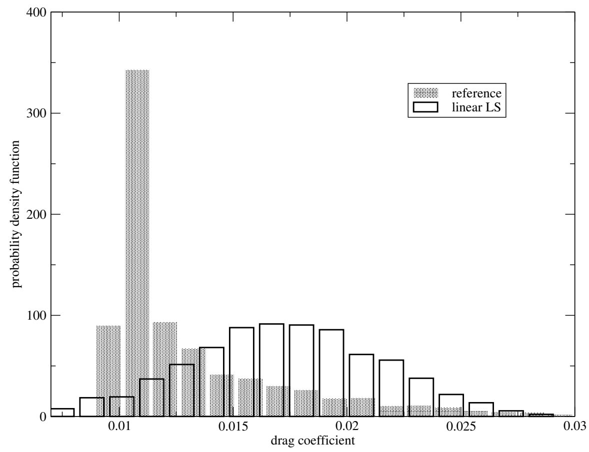

Figure 8: Probability density function for the drag coefficient: reference result compared to linear fitting.

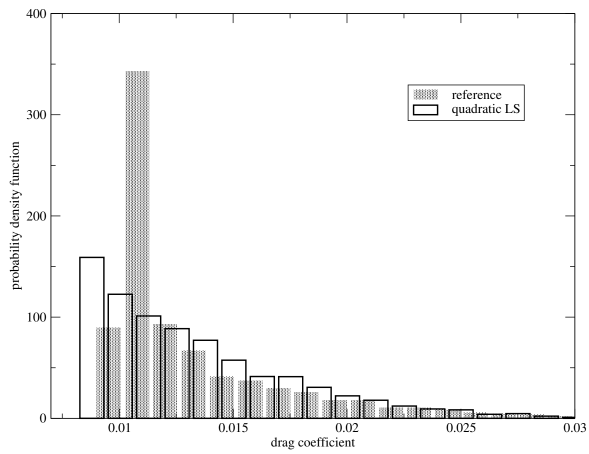

Figure 9: Probability density function for the drag coefficient: reference result compared to quadratic fitting.

tween the RBFs fitting and the CFD results at Mach number 0.82. To accurately represent the curvature in this region, the database must be enlarged. Using $N_c = 7$ points in the database provides satisfactory results (figures 12 and 13). This choice is adopted for the next computations. Table 2 compiles the statistics obtained for the different cases.

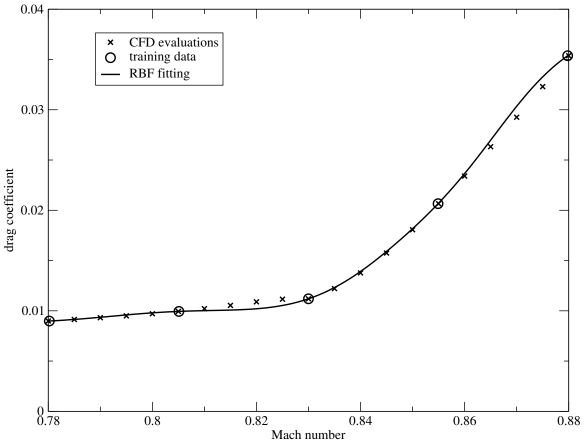

Figure 10: Five-point fitting using RBFs.

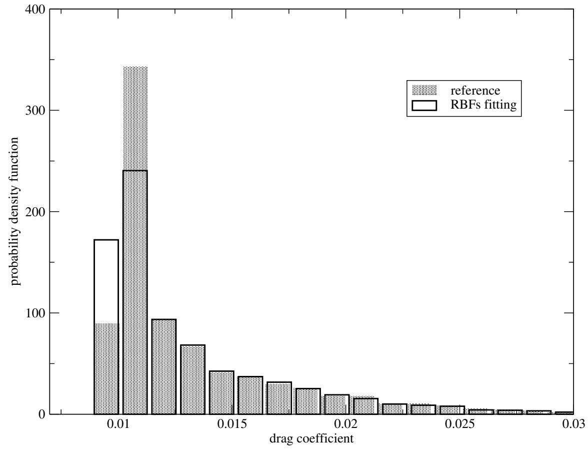

Figure 11: Probability density function for the drag coefficient: reference result compared to RBFs (5 points).

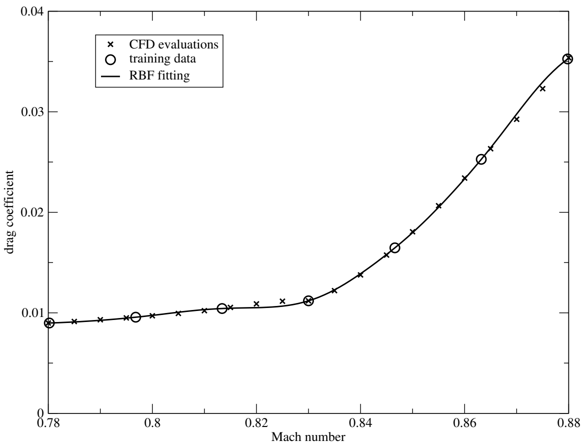

Figure 12: Seven-point fitting using RBFs.

Case	Mean	Variance
Reference	0.013154	1.5658E-05
Linear fit (5 pts)	0.017229	1.7953E-05
Quadratic fit (5 pts)	0.013262	1.9482E-05
RBFs (5 pts)	0.013029	1.7229E-05
RBFs (7 pts)	0.013068	1.5899E-05

Table 2: Statistics for the drag obtained with the different methods.

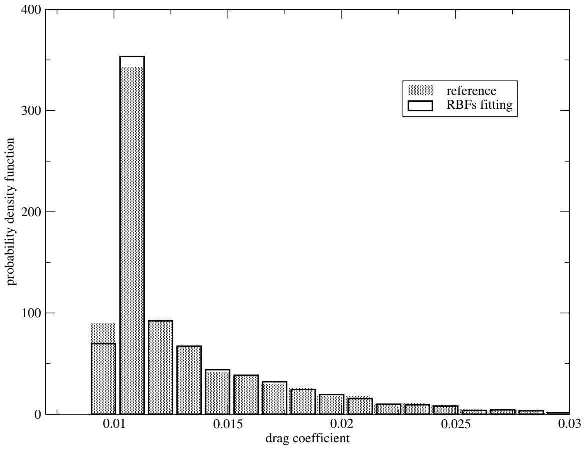

Figure 13: Probability density function for the drag coefficient: reference result compared to RBFs (7 points).

Robust design

Once uncertainty propagation relies on a safe basis, the robust optimization problem (9) can be solved. In the present study, the two-objective problem is addressed using a composite cost function, which is the weighted sum of the mean and variance. The optimization algorithm consists in a multi-level Particle Swarm Optimization (PSO) algorithm described in depth in [5]. Five optimization exercises are solved using different weights to draw the Pareto front (see figure 14). The results corresponding to these five optimization exercises are summarized in table 3.

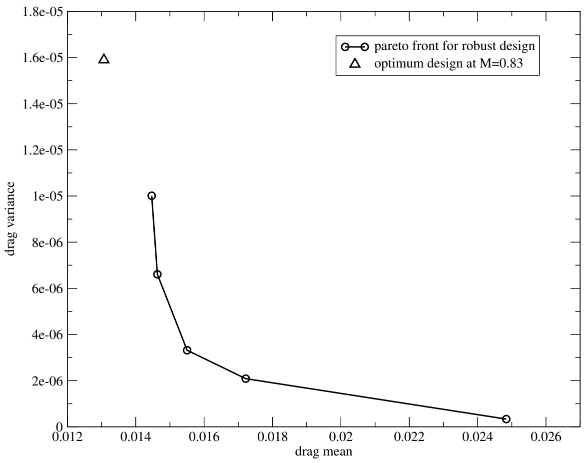

Figure 14: Pareto front for the two-objective robust design problem.

One can notice that the optimum design at nominal Mach number has a lower drag than the design found minimizing the mean drag. It is due to the fact that the optimum design at nominal Mach number does not respect the lift constraint in the probabilistic sense. For this design, the probability to reach the prescribed lift value is only 0.5057.

The evolution of the drag coefficient as the Mach number varies can be seen in figure 15 for three points on the Pareto front. These results are obtained by performing *a posteriori* twenty-one CFD analyses in the interval $[\mu_M - 3\sigma_M, \mu_M + 3\sigma_M]$ for the three designs under consideration. The corresponding PDFs are depicted on figure 16. As can be seen, a design is found, for which the drag is almost constant over the interval $[\mu_M - 3\sigma_M, \mu_M + 3\sigma_M]$. However,

Weight for the mean	Weight for the variance	Mean	Variance	Constraint probability
1.0	0.0	0.0144	1.001E-05	0.9504
0.75	0.25	0.0146	6.607E-06	0.9501
0.5	0.5	0.0155	3.316E-06	0.9504
0.25	0.75	0.0172	2.085E-06	0.9512
0.0	1.0	0.0248	3.355E-07	0.9576

Table 3: Statistics for the different robust optimization exercises.

its mean drag is poor, since it is not taken into account during the optimization. On the contrary, a second design optimizes the mean drag, but exhibits a significant drag increase for high Mach numbers. Finaly, a trade-off design is shown, for which the mean is slightly degraded but the drag fluctuations are moderate. The comparison of the PDFs clearly shows the characteristics of these different designs.

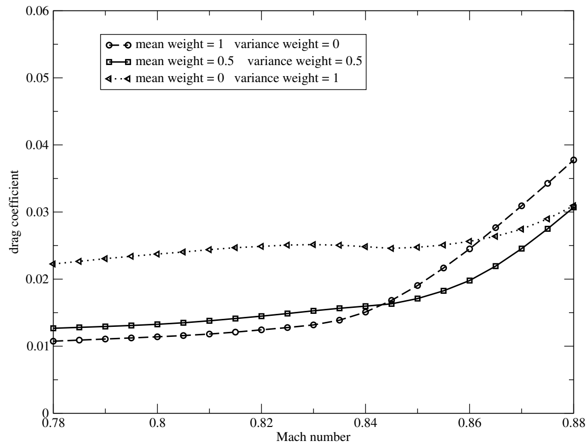

Figure 15: Drag variation for fluctuating Mach number: robust designs from the Pareto front.

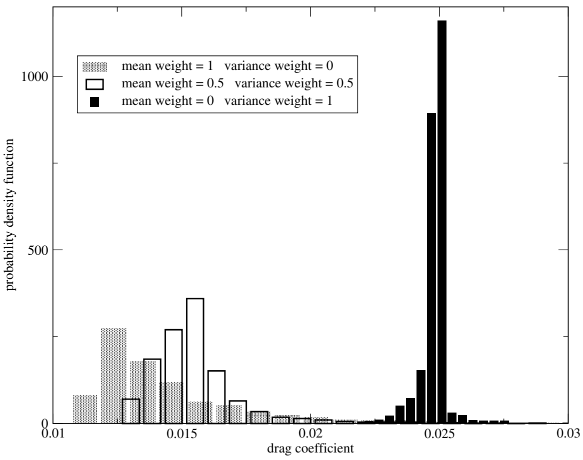

Figure 16: Probability density functions for the drag coefficient: robust designs from the Pareto front.

A comparison of the wing shapes at the root section is depicted in figure 17, for the optimum design at nominal Mach number and the robust design (trade-off between mean and variance). One can notice that the pressure side of the robust design is particularly flat.

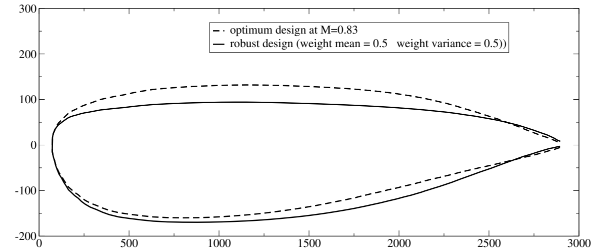

Figure 17: Comparison of the wing shape at the root section: optimum design at nominal Mach number and robust design.

CONCLUSION

To overcome the difficulty related to the high computational cost required by evolutionary computations or robust design, a two-level modeling strategy is proposed, that relies on the use of meta-models, such as radial basis functions.

This methodology is demonstrated for a realistic wing design in transonic regime. A genetic algorithm is accelerated using an inexact pre-evaluation approach and a robust design problem with uncertain Mach number is solved using an approximate statistics evaluation.

The proposed approach has been found particularly effective to reduce the computational cost without degrading the results.

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