Aerodynamic Wing Optimization for Propeller Driven Aircraft

B. R. RAKSHITH*, S. M. DESHPANDE*, C. PRAVEEN†, R. NARASIMHA*

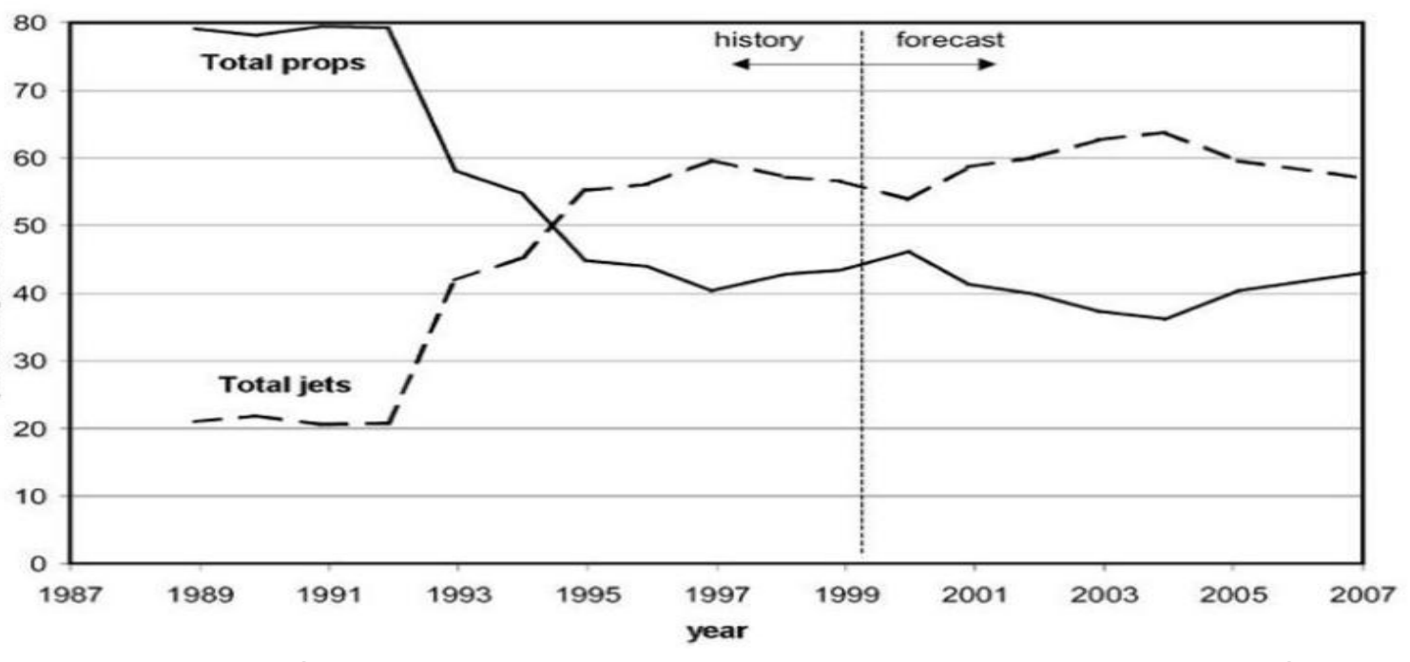
*Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India
†TIFR Centre for Applicable Mathematics, Bangalore, India

MOTIVATION

Turboprops are making a come back owing to:

- Climate change concerns
- Uncertain fuel prices
- Connectivity demands
- Crowding in the skies
- Advent of open rotors and other new technologies
- *THEIR INHERENTLY HIGHER FUEL EFFICIENCY*

Turboprops vs jets
(Number of Units)
Butterworth-Hayes, P.: Aerospace America January 1999



Decline in the usage of turboprops since 90's because of:

- Cheap oil
- Greater cabin noise and vibration
- More frequent flying in turbulent weather due to lower flight altitudes

OBJECTIVE

ATR 72-500
(https://www.atr.fr)

Bombardier Q-400
(http://www.bombardier .com)

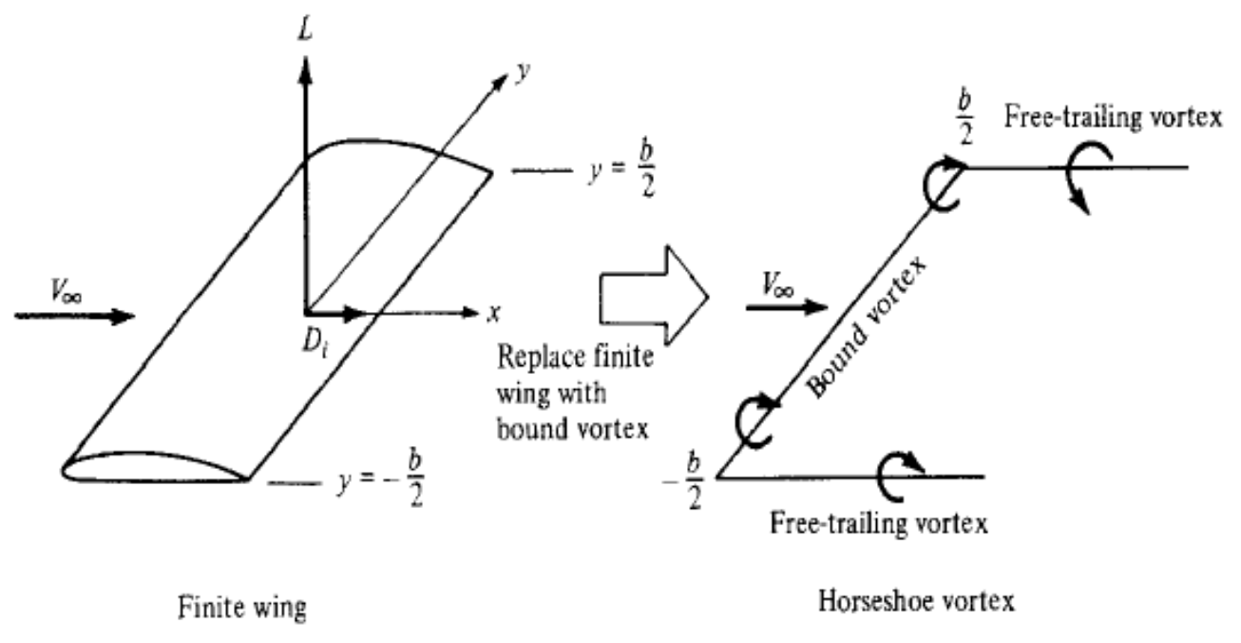
To design an integrated wing-propeller system so that the wing ***exploits*** propeller aerodynamic characteristics for achieving improved performance

SOME MAJOR EARLIER RESEARCH

- Prandtl, L., "***Mutual influence of wings and propellers***", NACA Technical Note 1921
- Koning, C., "***Influence of Propeller on other parts of the Airplane Structures***", Aerodynamic Theory, 1935
- Jameson, A., "***The Analysis of Propeller-Wing flow Interaction***", NASA-228, 1969.
- Kroo, I. "***Propeller-Wing Interaction for Minimum Induced Loss***", J.Aircraft, July 1986.
- Veldhuis, L.L.M. et al ***"Aerodynamic Optimization of wing in multi-engined tractor propeller arrangements***", Aircraft Design, 2000.

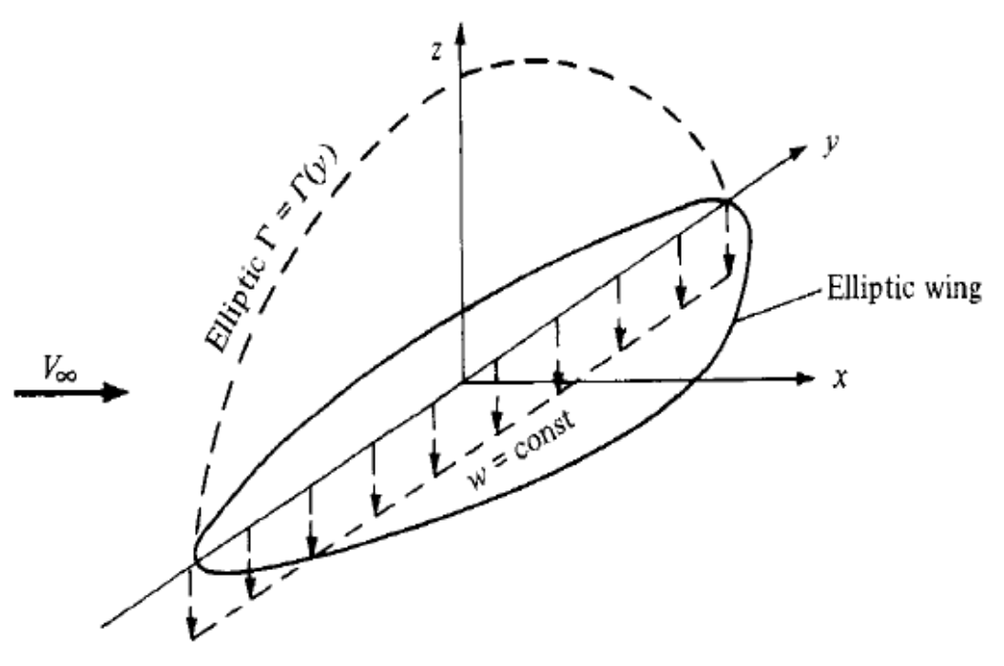
LIFTING LINE THEORY

Replacement of a finite wing with bound vortex



Fundamentals of Aerodynamics – John D. Anderson

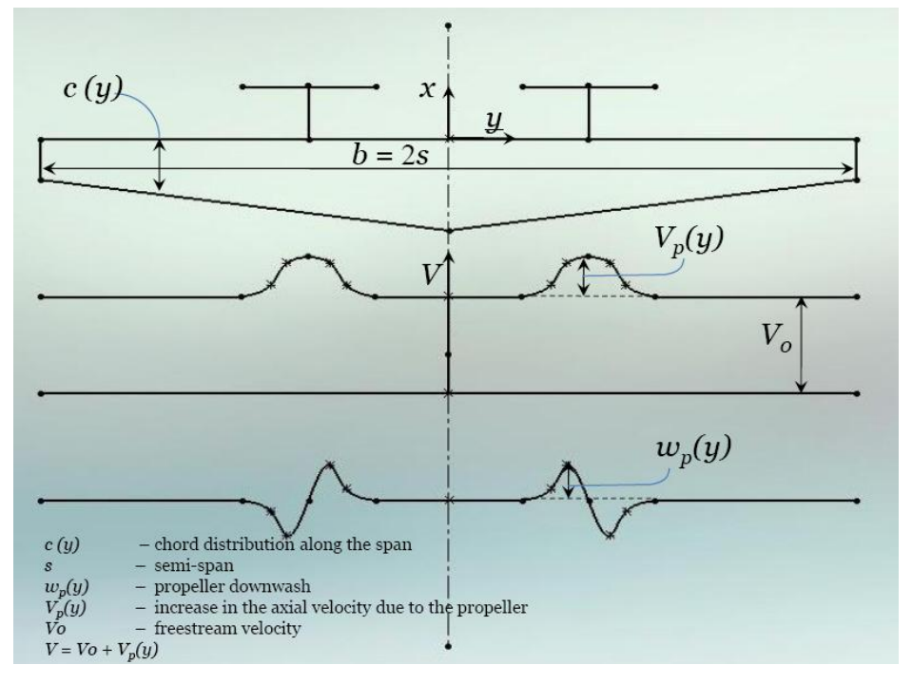
Elliptic load distribution

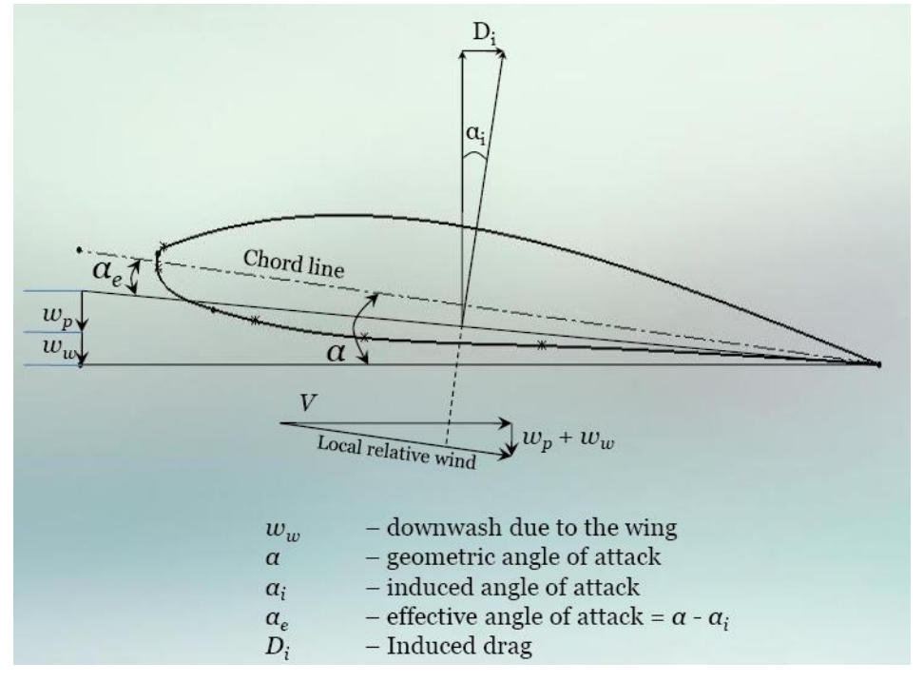


Fundamentals of Aerodynamics – John D. Anderson

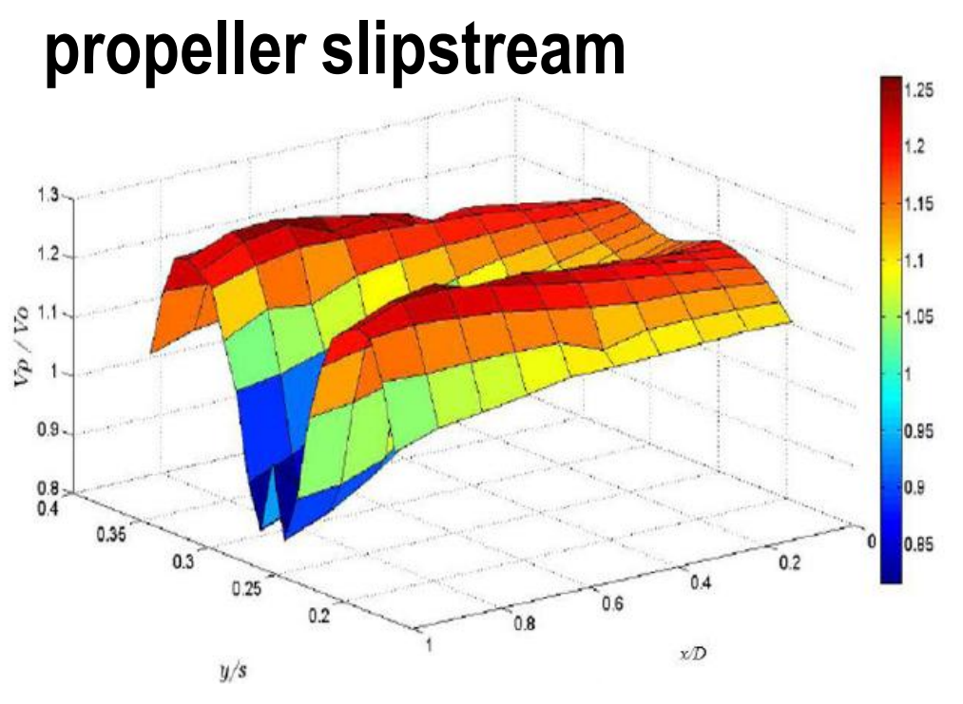
Spitfire
(http://www.images.google.co.in)

EXTENDED LIFTING LINE THEORY

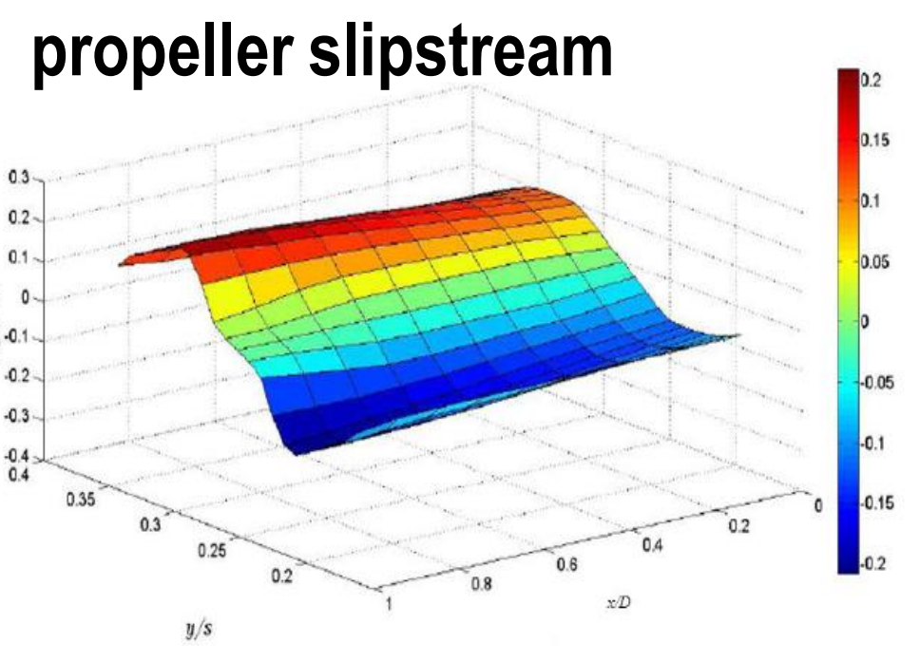




Plot of axial velocity in the propeller slipstream



Plot of downwash in the propeller slipstream



(Flow field computed by Mr. Josy P. Pullockara, CTFD, NAL, Bangalore, India)

EXTENDED LIFTING LINE THEORY

Given:

$$V(y) = V_0 + V_p(y)$$
$$w_p(y)$$

Total downwash:

$$\begin{aligned} w(y) &= w_w(y) + w_p(y) \\ &= \frac{1}{4\pi}\int_{-s}^{+s}\frac{1}{y-y'}\frac{d\Gamma}{dy'}dy' + w_p(y) \end{aligned}$$

Circulation:

$$\begin{aligned} \Gamma(y) &= c_l(y)c(y)V(y) \\ &= a_o c(y)V(y)\left(\alpha - \frac{w_w(y)+w_p(y)}{V(y)}\right) \end{aligned}$$

Lift:

$$L = \int_{-s}^{+s}\rho V(y)\Gamma(y)dy$$

Drag:

$$D_i = \int_{-s}^{+s}\rho w(y)\Gamma(y)dy$$

METHOD OF SOLUTION

Following Glauert - 1926

$$y = -s\cos\theta \qquad \theta \text{ varies from 0 to } \pi$$

$$\Gamma = 4sV_o\sum_{k=1}^{\infty}A_k\sin k\theta$$

Fundamental equation to determine Fourier coefficients :

$$\left(\sum_{k=1}^{\infty}A_k\sin k\theta\right)(k\mu+\sin\theta) = \mu\frac{V(\theta)}{V_o}\sin\theta\left(\alpha - \frac{w_p(\theta)}{V(\theta)}\right);\ \ \mu = \frac{a_o c}{8s}$$

Lift (*L*):

$$L = 4\rho V_o s^2\int_0^{\pi}V(\theta)\sin\theta\sum_{k=1}^{\infty}A_k\sin k\theta d\theta$$

Drag (*D_i*):

$$D_i = 2\pi s^2\rho V_o^2\sum_{k=1}^{\infty}kA_k^2 + A_kB_k$$

$$B_k = \frac{2}{\pi}\int_0^{\pi}\frac{w_p(y)}{V_o}\sin k\theta\sin\theta$$

NUMERICS

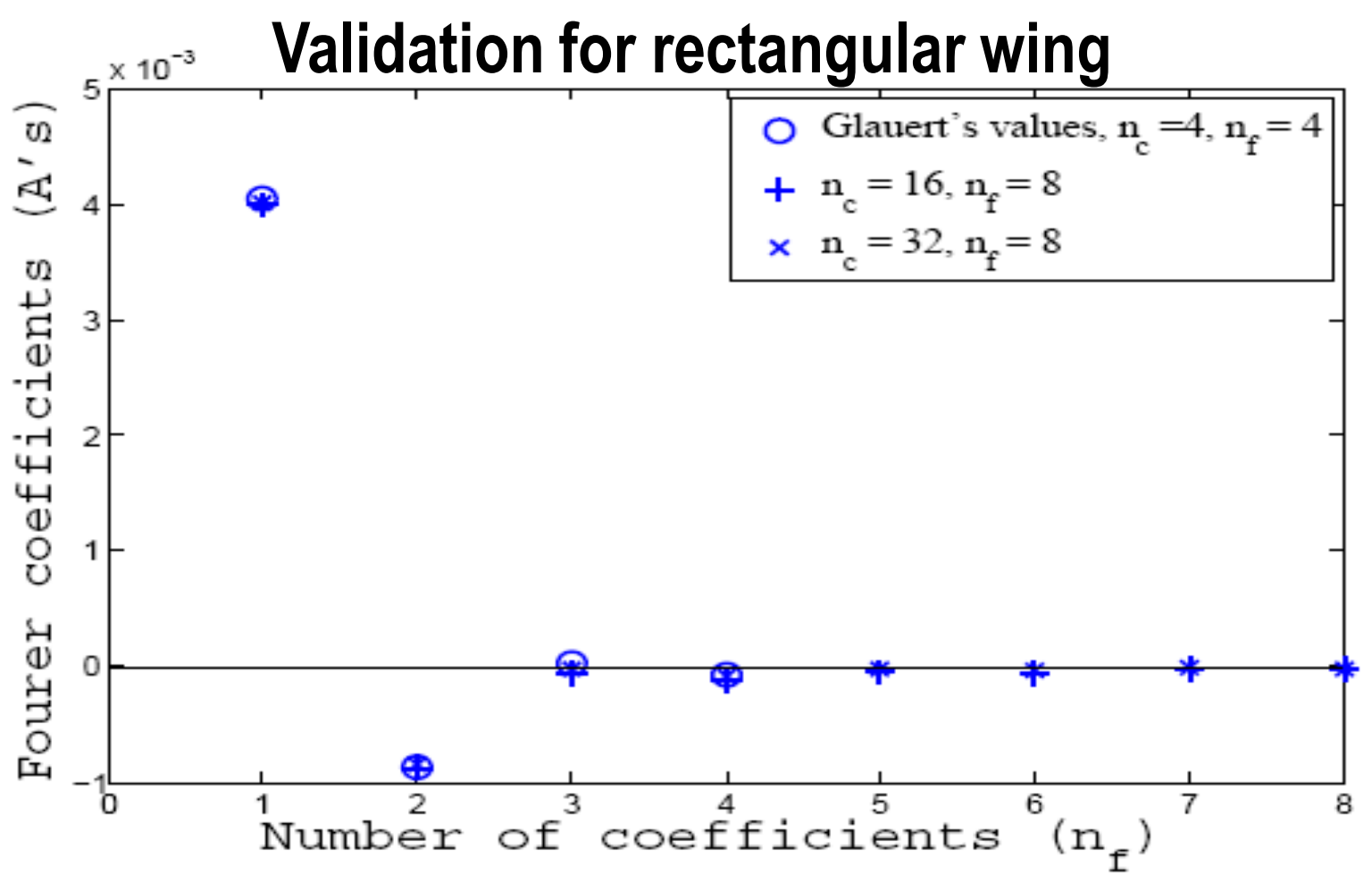
Solution by method of collocation using least squares approach:

- **Select number of collocation points (n_c)**
- **Select number of Fourier modes (n_f)**
- $n_c > n_f$

$$\mathrm{LL}(\theta, A_1....A_{n_f}) = \left(\sum_{k=1}^{n_f}A_k\sin k\theta\right)(k\mu+\sin\theta) - \mu\frac{V(\theta)}{V_o}\sin\theta\left(\alpha - \frac{w_p(\theta)}{V(\theta)}\right) = 0$$

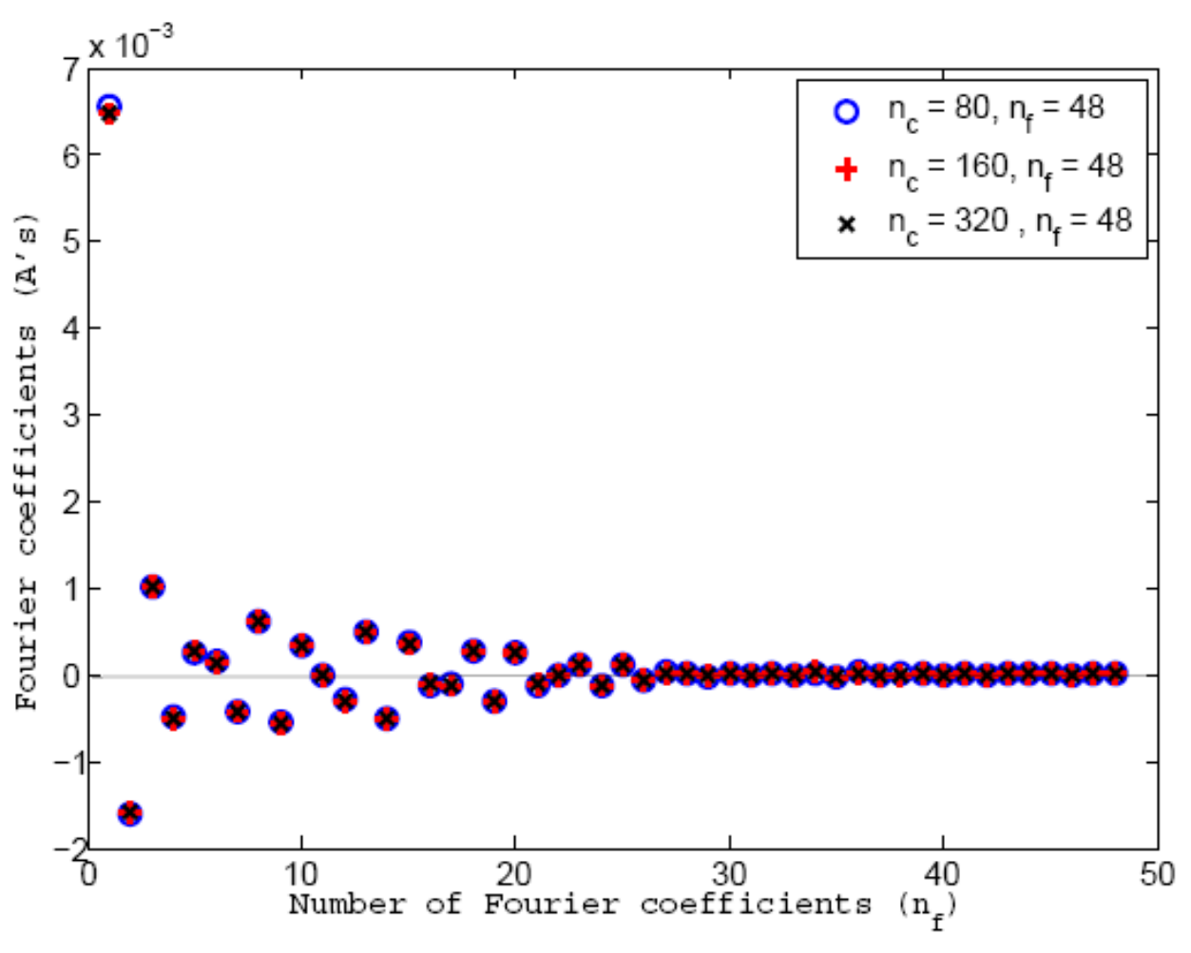
$$\frac{\partial}{\partial A_j}\sum_{i=1}^{n_c}\left[\mathrm{LL}(\theta_i, A_1...A_{n_f})\right]^2 = 0\ ;\quad j = 1......n_f$$

Validation for rectangular wing

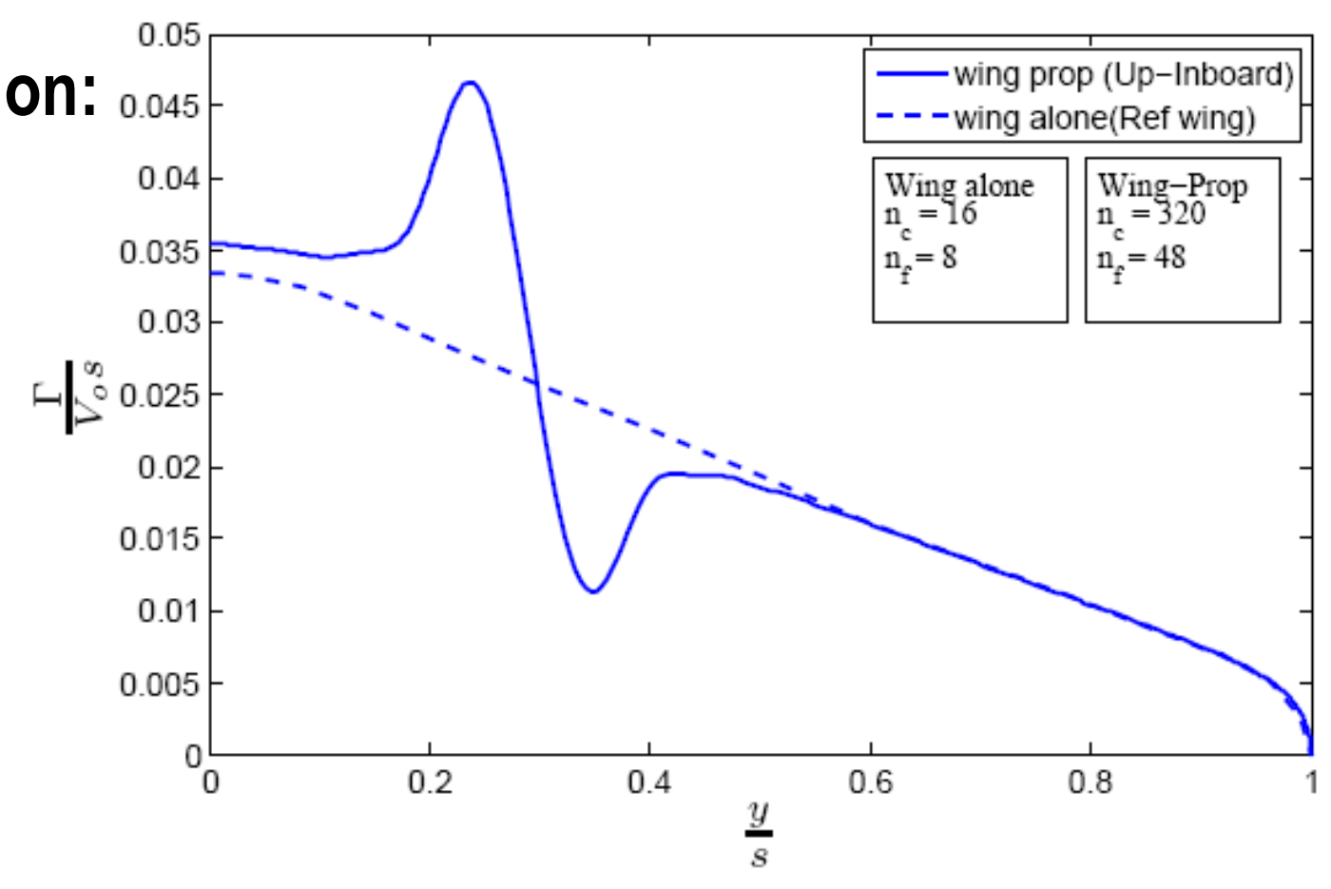


WING-PROPELLER CONFIGURATION

Convergence for wing-prop:



Circulation distribution:



OPTIMIZATION

Minimise total drag (induced + profile) for wing-propeller configuration wrt chord and twist distribution

Cost function:

$$\begin{aligned} J &= D_i + D_p \\ &= J(A_1...A_{n_f}, c_1...c_{n_c}, \alpha_{t1}...\alpha_{tn_c}) \\ D_p &= \int_{-s}^{s}\frac{1}{2}\rho c_{dp}V(y)^2c(y)dy \end{aligned}$$

- D_i Induced drag
- D_p Profile drag
- c_{dp} is obtained by c_l vs c_{dp} data of selected aerofoil section at each spanwise station
- Could include other parameters for e.g. wing root bending moment

Constraints

- Aerodynamic
  - Lifting-Line equation (LL = 0)
  - Lift (L)
  - ...
- Geometric
  - Wing area (S)
  - Root chord (c_m)
  - Tip chord (c_t)
  - Bounds on twist
  - ...
- Other
  - Structural
  - ...

Control vectors

- chords ($c_1...c_{n_c}$)
- twist ($\alpha_{t1}...\alpha_{tn_c}$)
- ...

SHAPE PARAMETERIZATION

Why Parameterization?

- Computationally relatively inexpensive
- Optimal designs can be smooth

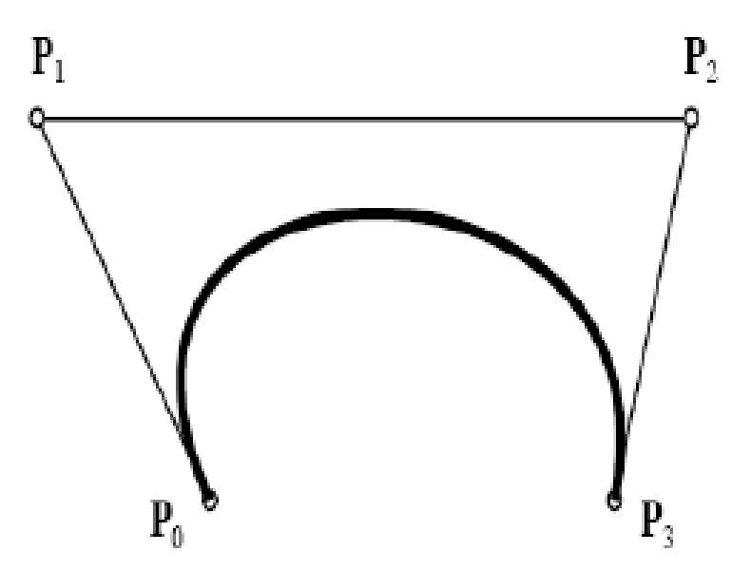
Types of Parameterization:

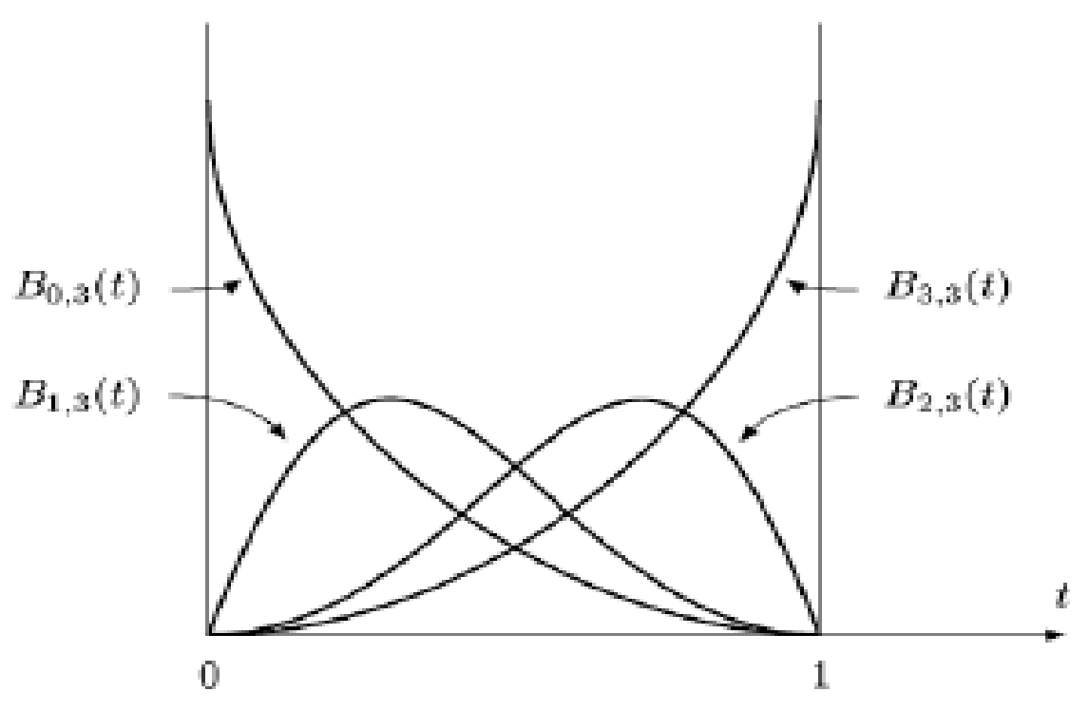
- Bezier curves
- Splines
- ...

Bezier curves:

$$\begin{aligned} c(y) &= \sum_{i=0}^{n}w_iB_i(y) \\ B_i(y) &= \binom{n}{i}y^i(1-y)^{n-i};\quad 0 \le y \le 1 \\ n_w &= n+1 \end{aligned}$$

- $B_i(y)$ - Bernstein polynomials
- n_w - number of control points
- w_i - Control points





PROWING

Optimizes the wing design using a constrained optimizer fmincon of MATLAB

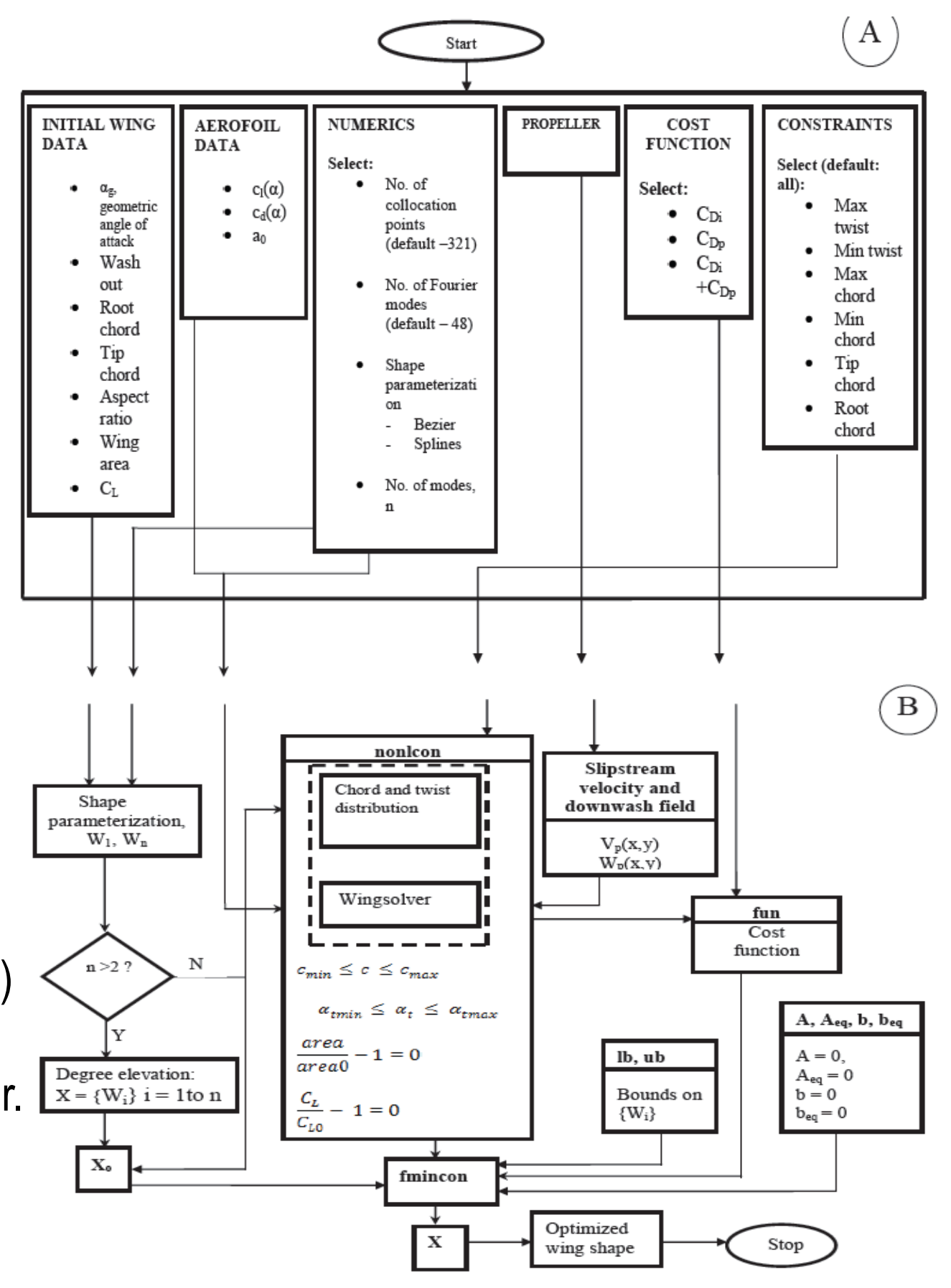
fmincon:
Performs constrained minimization of a nonlinear function

$$\min_x f(x) = 0$$

Subject to:

$$c(x) \le 0$$
$$ceq = 0$$
$$lb \le x \le ub$$

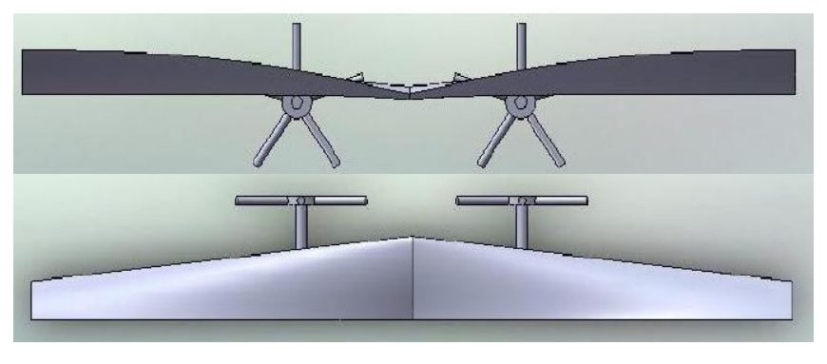
x, lb, ub are vectors, c(*x*) and ceq(*x*) are functions that return vectors and *f(x)* is a function that returns a scalar.



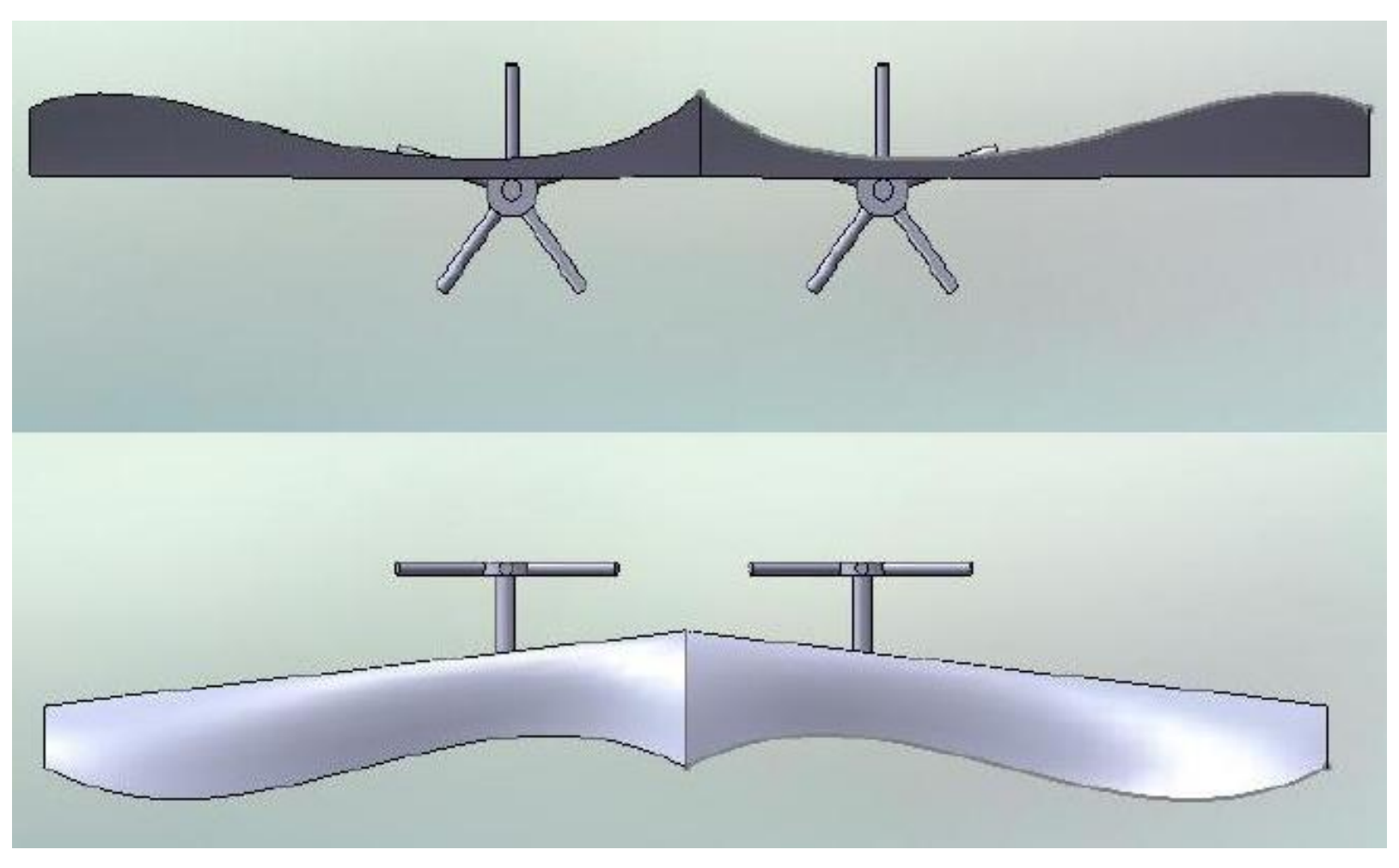
Results of optimization are validated against Prandtl's result

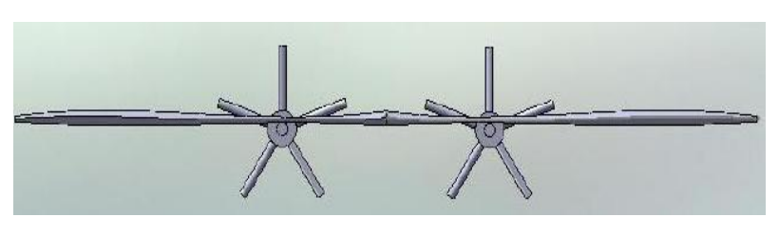
OPTIMIZED WING (1)

Reference Wing



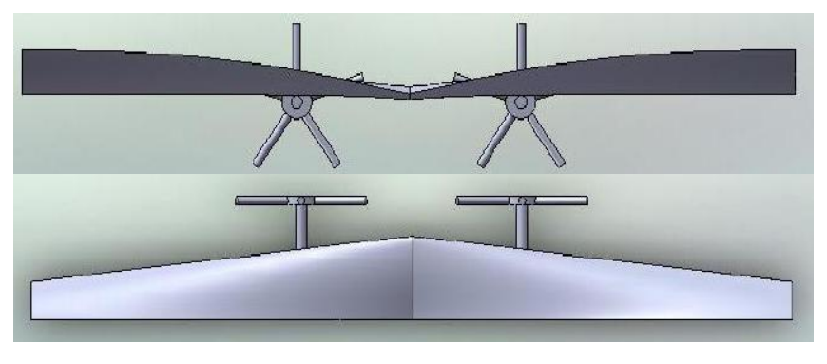
n_c = 320, n_f = 48, n_{wc} = 6, n_{wt} = 4, −14° t 14°, C_L = 0.27, twist magnified by 20 times!!!, AR = 12, c_m = 0.2116s, c_t = 0.0952s, **ΔC_d = - 8.74%**, **ΔC_{di} = - 19.27%**, Up-Inboard



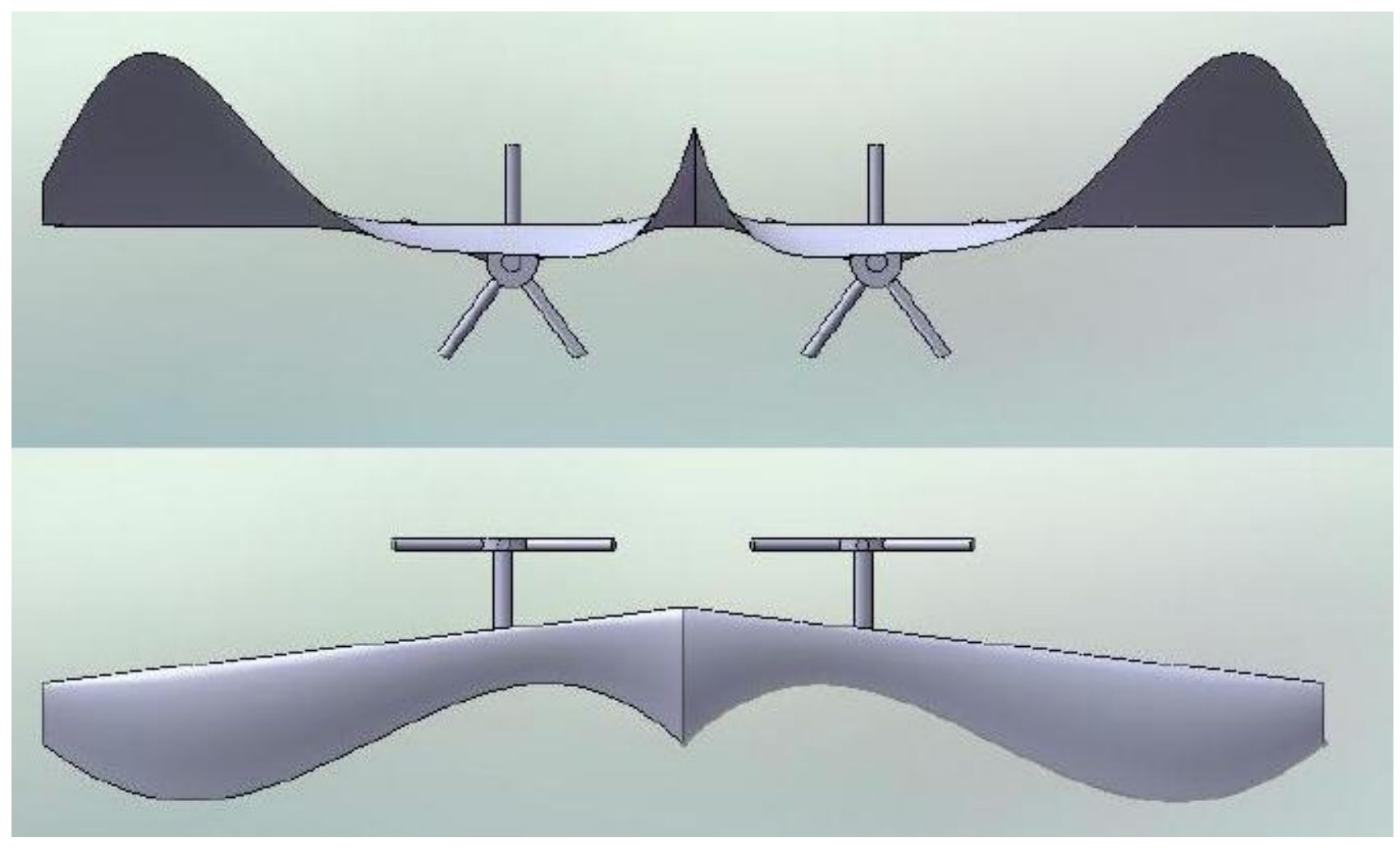


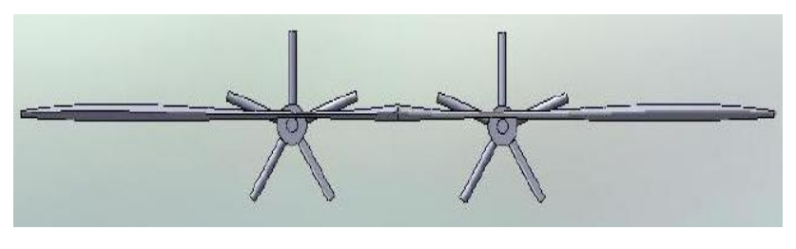
OPTIMIZED WING (2)

Reference Wing



n_c = 320, n_f = 48, n_{wc} = 6, n_{wt} = 4, −14° t 14°, C_L = 0.27, twist magnified by 20 times!!!, AR = 12, c_m = 0.2116s, c_t = 0.0952s, **ΔC_d = - 13.86%**, **ΔC_{di} = - 32.07%**, Up-Inboard





CONCLUSION

- Current turboprops have wings which do not **fully exploit** the propeller slipstream.
- #Optimized wings are characterized generally by lower wing chords behind each propeller and higher wing chords elsewhere including one or more of the following areas :
  - **Outboard of the propeller, towards the wing tip**
  - **Inboard of the propeller, towards the fuselage**
  - **Between propellers if there is more than one on each side of the wing**

#Patents pending

FUTURE WORK

- Validation of results through CFD
- Effect of wing on the propeller

ACKNOWLEDGEMENTS

- Dr. V. Y. Mudkavi and Mr. Josy P. Pullockara, CTFD, NAL, Bangalore, India