

Computational Methods for PDE

Assignment I

12 January 2013

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Problem 1 Consider the function

$$f(x) = \sin(x)$$

At $x = \pi/4$, compute the first derivative approximation using forward difference and central difference scheme for different values of h . Start with $h = 1/100$ and reduce h by a factor of two every time, until you reach $h \approx 10^{-14}$. You will need to divide h about 40 times to reach this level. Plot the absolute value of the error in the derivative approximation as a function of h on a log-log plot. You should get a figure like that shown below. The error starts increasing at some point because of round-off errors. Can you try to explain this mathematically? In the process, try to find the optimum value of h . Note that matlab does all computations in double precision, the machine precision being about 10^{-14} . Compute the convergence rate using the formula

$$p = \frac{\log \|e_h\| / \|e_{h/2}\|}{\log(2)}$$

and verify the first order and second order accuracy.

You can write the programs in matlab, fortran, C or C++.

