

Computational Methods for PDE

Assignment 2

21 January 2013

Submit by 1 February 2013

Problem 1 Consider the following BVP

$$-u''(x) = f(x) \quad x \in (0, 2\pi)$$

with boundary conditions

$$u(0) = u(2\pi) = 0$$

Consider the following finite difference scheme

$$-D_x^+ D_x^- U_j = f_j + \frac{h^2}{12} D_x^+ D_x^- f_j, \quad j = 2, 3, \dots, N-1$$

and

$$U_1 = U_N = 0$$

Compute the local truncation error of the above scheme and show that it is $\mathcal{O}(h^4)$. Hence show that the scheme is fourth order accurate.

Take $f(x) = \sin(x)$ so that the exact solution is $u(x) = \sin(x)$. Write a computer program to implement the above scheme. Solve the problem for $N = 10, 20, 40, 80, 160, 320$ grid points and compute error in maximum norm and discrete L^2 norm in each case. Plot the error versus N on a log-log plot and verify the fourth order accuracy in both the norms.

You can write the programs in matlab, fortran, C or C++.