Computational Methods for PDE

Assignment 3

4 February 2013
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Problem 1 *Consider the following IBVP in* $\Omega = (0,1) \times (0,1)$

$$u_t = u_{xx} + u_{yy} + F(x,y,t) \qquad (x,y) \in \Omega, \quad t > 0 \tag{1}$$

$$u(x,y,0) = f(x,y), \qquad (x,y) \in \Omega \tag{2}$$

$$u(x,y,t) = g(x,y,t), \qquad (x,y) \in \partial\Omega, \qquad t > 0 \tag{3}$$

Take the exact solution to be

$$u(x,y,t) = \frac{1}{1 + 50(x - x_0(t))^2 + 50(y - y_0(t))^2}$$

where

$$x_0(t) = \frac{1}{2} + \frac{1}{4}\cos(2\pi t), \qquad y_0(t) = \frac{1}{2} + \frac{1}{4}\sin(2\pi t)$$

and obtain F, f, g *from this exact solution. Solve the problem using the Peaceman-Rachford scheme on a grid of* $M_x \times M_y$ *points*

$$(1 - \frac{1}{2} r_x \delta_x^2) u^{n+\frac{1}{2}} = (1 + \frac{1}{2} r_y \delta_y^2) u^n + \frac{\Delta t}{2} F^{n+\frac{1}{2}} \tag{4}$$

$$(1 - \frac{1}{2} r_y \delta_y^2) u^{n+1} = (1 + \frac{1}{2} r_x \delta_x^2) u^{n+\frac{1}{2}} + \frac{\Delta t}{2} F^{n+\frac{1}{2}} \tag{5}$$

Use the boundary condition for $u^{n+\frac{1}{2}}$ *as*

$$u_{j,k}^{n+\frac{1}{2}} = g(x_j, y_k, t^n + \frac{\Delta t}{2}), \qquad j = 1, M_x$$

Solve the problem upto a final time of $T = 2$ *with* $\Delta t = 0.01$ *and meshes of size* 50×50 *and* 100×100.

For the programming, define following functions

- `function u = ExactSolution(x,y,t)`
- `function f = SourceTerm(x,y,t)`

Create the tridiagonal matrices using `spdiags` function and use the matlab \ operator to solve the tridiagonal matrix problems. Here is a rough sketch of the program.

```
Set parameters
Allocate memory
Create mesh
Create the two tridiagonal matrices
Set initial condition
While t < T
   Set boundary conditions on unph
   do k=2:My-1
      create rhs vector
      Solve to get unph(2:Mx-1, k)
   end
   Set boundary conditions on unp1
   do j=2:Mx-1
      create rhs vector
      Solve to get unp1(j,2:My-1)
   end
   t = t + dt
   Plot solution
end
```

In the class, I started indices `j, k` from zero. You have to adjust this to start from one in the matlab program.

You can write the programs in matlab, fortran, C or C++.