Appendix A

Euler test cases

A.1 1-D: linear advection

If the initial velocity and pressure are constant, then they remain constant for future times. The density is advected at the constant velocity. So an initial condition

$$\rho(x,0) = f(x), \qquad u(x,0) = u_0, \qquad p(x,0) = p_0$$

has exact solution

$$\rho(x,t) = f(x - u_0 t), \qquad u(x,t) = u_0, \qquad p(x,t) = p_0$$

Take $f(x)$ to be a periodic function and use periodic boundary conditions. E.g.

$$f(x) = 1 + \frac{1}{2}\sin(2\pi x), \qquad x \in [0,1]$$

with $u_0 = p_0 = 1$ and $\gamma = 1.4$.

A.2 1-D: Sod test

This is a Riemann problem [4] given in Table (A.1) which develops a left rarefaction, a contact wave and a right shock. Solve this on a domain $[0,1]$ with initial jump at $x = 0.5$, $\gamma = 1.4$ and upto time $t = 0.2$ using Neumann boundary conditions.

	ρ	u	p
$x < 0.5$	1.0	0.0	1.0
$x > 0.5$	0.125	0.0	0.1

Table A.1: Initial conditions for Sod test case

	ρ	u	p
$x < 0.3$	1.0	0.75	1.0
$x > 0.3$	0.125	0.0	0.1

Table A.2: Initial conditions for modified Sod test case with sonic rarefaction

	ρ	u	p
$x < -4$	3.857143	2.699369	10.33333
$x > -4$	$1.0 + 0.2\sin(5x)$	0.0	1.0

Table A.3: Initial conditions for Shu-Osher test case

A.3 1-D: Sod test with sonic rarefaction

This is a modification of Sod test case where the flow reaches sonic state inside the expansion fan and is a good test to check entropy condition satisfaction. The domain is $[0, 1]$ and initial conditions are given in Table (A.2). The solution is computed up to time $t = 0.2$ with $\gamma = 1.4$.

A.4 1-D: Shu-Osher test case

An initial jump separates a constant state on the left with a smooth density profile on the right. The solution involves interaction of a shock with a smooth profile and hence is challenging to get accurate solutions. Solve this on a domain $[-5, +5]$ with initial jump at $x = -4$ upto time $t = 1.8$. Use $\gamma = 1.4$ and Neumann boundary conditions. The initial condition is given in Table (A.3).

A.5 1-D: 123 problem

The initial condition is given by [6]

$$(\rho, u, p) = \begin{cases} (1.0, \ -2.0, \ 0.4) & x < 0.5 \\ (1.0, \ +2.0, \ 0.4) & x > 0.5 \end{cases}$$

The computational domain is $[0, 1]$ and the final time is $t = 0.15$. The density becomes very small in the middle of the domain and can be challenging test in terms of maintaining positive density.

A.6 1-D: Interaction of blast waves

The problems involves interaction of two shock waves and the initial condition is given by [8]

$$(\rho, u, p) = \begin{cases} (1.0,\ 0.0,\ 1000.0) & x < 0.1 \\ (1.0,\ 0.0,\ 0.01) & 0.1 < x < 0.9 \\ (1.0,\ 0.0,\ 100.0) & x > 0.9 \end{cases}$$

The domain can be taken as $[0, 1]$ and the computations performed upto the time $t = 0.038$. Solid wall boundary conditions are used on both end points of the domain.

A.7 2-D: Isentropic vortex

The flow consists of a vortex that moves with a constant speed [9]. The initial condition is given by

$$\begin{aligned} \rho &= \left[1 - \frac{(\gamma-1)\beta^2}{8\gamma\pi^2}\exp(1-r^2)\right]^{1/(\gamma-1)} \\ u &= M\cos\alpha - \frac{\beta}{2\pi}\exp(\tfrac{1}{2}(1-r^2))(y-y_0) \\ v &= M\sin\alpha + \frac{\beta}{2\pi}\exp(\tfrac{1}{2}(1-r^2))(x-x_0) \\ p &= \rho^\gamma \end{aligned}$$

where

$$r^2 = (x-x_0)^2 + (y-y_0)^2$$

Here (x_0, y_0) is the center of the vortex. The vortex moves at an angle α to the x axis at a constant speed of M. The flow has constant entropy everywhere and for all times.

Take a domain of $[-5, +5] \times [-5, +5]$ with these parameters

$$(x_0, y_0) = (0, 0), \quad \beta = 5, \quad M = 0.5, \quad \alpha = \pi/4$$

Use periodic boundary conditions and run up to a time of $t = 10\sqrt{2}/M$; the vortex would have come back to its initial position. Note that for high order methods, it is better to use a larger domain since the exponential functions may not have decayed at the boundary.

A.8 2-D: Shock reflection

This problem consist of a shock which hits the bottom wall and gets reflected and the flow eventually reaches a steady state. The domain is $[0, 4] \times [0, 1]$ and the initial condition can be taken as

$$\rho = 1, \qquad (u, v) = (2.9, 0.0), \qquad p = 1/\gamma$$

with $\gamma = 1.4$. The flow enters the domain from the left and top sides. The conditions at the left side are same as the above initial conditions while the conditions on the top are

$$\rho = 1.69997, \qquad (u, v) = (2.61934, -0.50632), \qquad p = 1.52819$$

The botton side is a solid wall and we can use Neumann conditions on the right side.