

# FEniCS Course

## Lecture 1: Introduction to FEniCS

*Contributors*

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FENICS  
PROJECT

What is FEniCS?

# FEniCS is an automated programming environment for differential equations

- C++/Python library
- Initiated 2003 in Chicago
- 1000–2000 monthly downloads
- Part of Debian and Ubuntu
- Licensed under the GNU LGPL



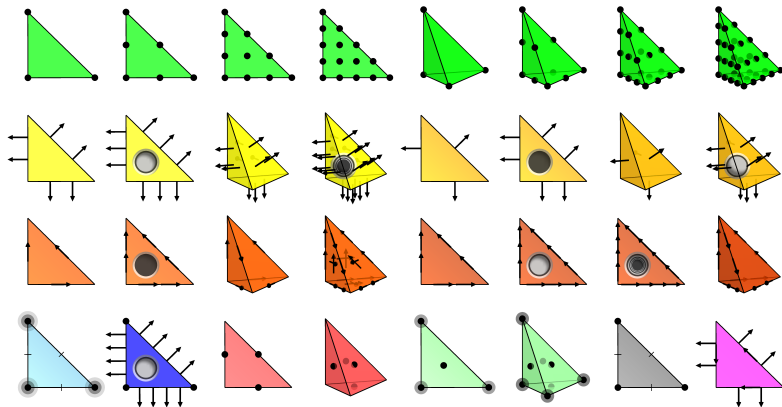
<http://fenicsproject.org/>

## Collaborators

*Simula Research Laboratory, University of Cambridge,  
University of Chicago, Texas Tech University, KTH Royal  
Institute of Technology, ...*

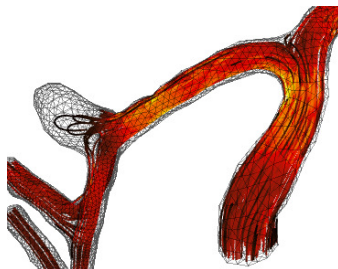
# FEniCS is automated FEM

- Automated generation of basis functions
- Automated evaluation of variational forms
- Automated finite element assembly
- Automated adaptive error control



What has FEniCS been used for?

# Computational hemodynamics



- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\begin{aligned}\dot{u} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) &= f \\ \nabla \cdot u &= 0\end{aligned}$$

# Computational hemodynamics (contd.)



```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T) -
        w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return 0.5*(grad(v) + grad(v).T)

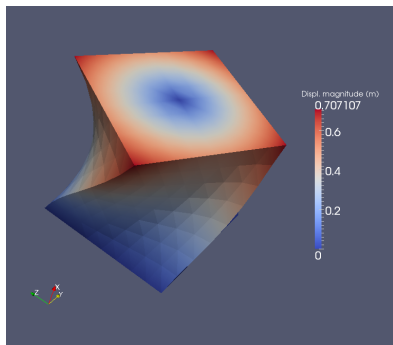
# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx +
    rho*inner(v, grad(u0)*(u0 - w))*dx \
    + inner(epsilon(v), sigma(U, p0))*dx \
    + inner(v, p0*n)*ds - mu*inner(grad(U).T*n,
        v)*ds \
    - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx -
    q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 -
    p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solvers to be implemented in a minimal amount of code

# Hyperelasticity



```
class Twist(StaticHyperelasticity):

    def mesh(self):
        n = 8
        return UnitCube(n, n, n)

    def dirichlet_conditions(self):
        clamp = Expression(("0.0", "0.0",
                             "0.0"))
        twist = Expression(("0.0",
                             "y0 + (x[1]-y0)*cos(theta) - (x[2]-z0)*sin(theta) - x[1]",
                             "z0 + (x[1]-y0)*sin(theta) + (x[2]-z0)*cos(theta) - x[2]"))
        twist.y0 = 0.5
        twist.z0 = 0.5
        twist.theta = pi/3
        return [clamp, twist]

    def dirichlet_boundaries(self):
        return ["x[0] == 0.0", "x[0] == 1.0"]

    def material_model(self):
        mu = 3.8461
        lmbda =
            Expression("x[0]*5.8+(1-x[0])*5.7")

        material = StVenantKirchhoff([mu,
                                       lmbda])
        return material

    def __str__(self):
        return "A cube twisted by 60 degrees"
```

- CBC.Solve is a collection of FEniCS-based solvers developed at CBC
- CBC.Twist, CBC.Flow, CBC.Swing, CBC.Beat, ...



How to use FEniCS?

# Hello World in FEniCS: problem formulation

## Poisson's equation

$$\begin{aligned} -\Delta u &= f && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega \end{aligned}$$

## Finite element formulation

Find  $u \in V$  such that

$$\underbrace{\int_{\Omega} \nabla u \cdot \nabla v \, dx}_{a(u,v)} = \underbrace{\int_{\Omega} f v \, dx}_{L(v)} \quad \forall v \in V$$

# Hello World in FEniCS: implementation

```
from dolfin import *

mesh = UnitSquare(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

u = Function(V)
solve(a == L, u, bc)
plot(u)
```

# Basic API

- Mesh Vertex, Edge, Face, Facet, Cell
  - FiniteElement, FunctionSpace
  - TrialFunction, TestFunction, Function
  - `grad()`, `curl()`, `div()`, ...
  - Matrix, Vector, KrylovSolver, LUSolver
  - `assemble()`, `solve()`, `plot()`
- 
- Python interface generated semi-automatically by SWIG
  - C++ and Python interfaces almost identical

Sounds great, but how do I find my way through the jungle?



# Three survival advices



Use the right Python  
tools



Explore the  
documentation



Ask, report and  
request

Use the right Python tools!

# Python tools

## Doc tools

- Standard terminal:  
    `> pydoc dolfin`  
    `> pydoc dolfin.Mesh`
- Python console  
    `>>> help(dolfin)`  
    `>>> help(dolfin.Mesh)`

## Sophisticated Python environments

**IDLE** the official (but rather limited) Python IDE

**IPython** <http://ipython.org/>  
provides a Python shell and notebook including syntax highlighting, tab-completion, object inspection, debug assisting, history ...

**Eclipse** plugin <http://pydev.org/>  
includes syntax highlighting, code completion, unit-testing, refactoring, debugger ...



# IPython notebook

IP[y]: Notebook

solving-poisson

Save QuickHelp

Actions

Download Print

Cell

Format Code Markdown

Output

Toggle Clear All

Insert

Above Below

Move

Up Down

Run

Selected All

Autosave: ☒

Kernel

Actions

Interrupt Restart

Kill kernel upon exit: ☐

Help

Links

Python IPython

NumPy SciPy

MPL SymPy

Shift-Enter: run selected cell

Ctrl-Enter: run selected cell in-place

Ctrl-m h: show keyboard shortcuts

Configuration

Tooltip on tab: ☒

Smart completer: ☒

Time before tooltip: 1200 milliseconds

Let's solve numerically the following variational problem: Find  $u \in H_0^1(\Omega)$  such that  $a(u, v) = L(v) \forall v \in H_0^1(\Omega)$  where  $a(u, v) = \int_{\Omega} \nabla u \nabla v \, dx$  and  $L(v) := \int_{\Omega} f v \, dx$ . To do that in FEniCS we start by defining a mesh:

```
In [26]: from dolfin import *
m = UnitSquare(10, 10)
print m

<Mesh of topological dimension 2 (triangles) with 121 vertices and 200 cells, ordered>
```

Now we need some function space. Let's take P1 elements

```
In [27]: V = FunctionSpace(m, "CG", 1)
```

It's time to define some test and trial functions.

```
In [28]: u = TrialFunction(V)
v = TestFunction(V)
```

And finally we can define the variational problem:

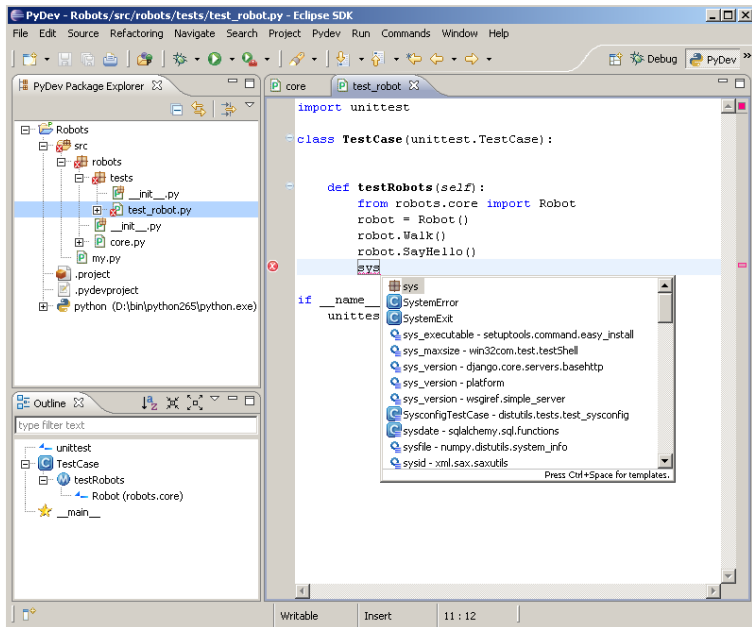
```
In [29]: a = inner(grad(u), grad(v)) * dx
f = Constant(1)
L = f * v * dx
def boundary(x, on_boundary):
    return on_boundary
u = Function(V)
zero = Constant(0)
bc = DirichletBC(V, zero, boundary)
solve(a == L, u, bc)
```

```
In [30]: plot(u)
```

```
Out[30]: <viper.viper_dolfin.Viper at 0x4b76890>
```

```
In [31]: interactive()
```

# Eclipse plugin Pydev



Explore the FEniCS documentation!

## Documentation for FEniCS 1.0.0

Our documentation includes a book, a collection of documented demo programs, and complete references for the FEniCS application programming interface (API). Note that the FEniCS API is documented separately for each FEniCS component. The most important interfaces are those of the C++/Python problem solving environment *DOLFIN* and the form language *UFL*.

(This page accesses the FEniCS 1.0.0 documentation. Not the version you are looking for? See [all versions](#).)

### The FEniCS Tutorial

A good starting point for new users is the *FEniCS Tutorial*. The tutorial will help you get quickly up and running with solving differential equations in FEniCS. The tutorial focuses exclusively on the FEniCS Python interface, since this is the simplest approach to exploring FEniCS for beginners.

### The FEniCS Book



*The FEniCS Book, Automated Solution of Differential Equations by the Finite Element Method*, is a comprehensive (700 pages) book documenting the mathematical methodology behind the FEniCS Project and the software developed as part of the FEniCS Project. The FEniCS Tutorial is included as the opening chapter of the FEniCS Book.

### The FEniCS Manual

*The FEniCS Manual* is a 200-page excerpt from the FEniCS Book, including the FEniCS Tutorial, an introduction to the finite element method and documentation of DOLFIN and UFL.

### Demos

A simple way to build your first FEniCS application is to copy and modify one of the existing demos:

---

Documented DOLFIN demos (Python)

---

Documented DOLFIN demos (C++)

The demos are *already installed on your system* or can be found in the `demo` directory of the DOLFIN source tree.

### Quick Programmer's References

Some of the classes and functions in DOLFIN are more frequently used than others. To learn more about these, take a look at the

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## Overview of Launchpad pages

This page contains links to all FENICS projects on Launchpad for convenience.

### Meta components

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<a href="#">FENICS Project</a>	<a href="#">Get Answers</a>	<a href="#">Report Bugs</a>	<a href="#">(Join Team and) Email Support</a>	<a href="#">Browse List Archives</a>

### Core components


<a href="#">DOLFIN</a>	<a href="#">Get Answers</a>	<a href="#">Report Bugs</a>	<a href="#">(Join Team and) Email Support</a>	<a href="#">Browse List Archives</a>
<a href="#">FFC</a>	<a href="#">Get Answers</a>	<a href="#">Report Bugs</a>	<a href="#">(Join Team and) Email Support</a>	<a href="#">Browse List Archives</a>
<a href="#">FIAT</a>	<a href="#">Get Answers</a>	<a href="#">Report Bugs</a>	<a href="#">(Join Team and) Email Support</a>	<a href="#">Browse List Archives</a>
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<a href="#">Dorsal</a>	<a href="#">Get Answers</a>	<a href="#">Report Bugs</a>	<a href="#">(Join Team and) Email Support</a>	<a href="#">Browse List Archives</a>

[http://fenicsproject.org/support/launchpad\\_pages.html](http://fenicsproject.org/support/launchpad_pages.html)

# Community help is centralized via launchpad



## DOLFIN

[Overview](#) [Code](#) [Bugs](#) [Blueprints](#) [Translations](#) **Answers**

### Questions for DOLFIN

DOLFIN > Questions

by relevancy

Search

Status

☒ Open ☒ Needs information ☒ Answered ☒ Solved ☐ Expired ☐ Invalid

Summary	Created	Submitter	Assignee	Status
201835 <a href="#">PeriodicBC crashing on coordinates I don't have</a>	2012-06-29	Stav Gold	—	Answered
201782 <a href="#">Error in gradient calculation along axes</a>	2012-06-29	K. Hoffmann	—	Open
201696 <a href="#">Properties of the goal function in non-linear problems</a>	2012-06-28	Daniel Bare	—	Solved
201639 <a href="#">listing dofs of a Facet</a>	2012-06-27	Nguyen Van Dang	—	Open
201638 <a href="#">Boundary conditions for more than one variable</a>	2012-06-27	minak	—	Answered
201548 <a href="#">Using PETSc command line options</a>	2012-06-26	M. Sussman	—	Solved

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DOLFIN

DOLFIN Team

<https://answers.launchpad.net/dolfin>

# Installation



Official packages for Debian and Ubuntu



Drag and drop installation on Mac OS X



Binary installer for Windows



Automated installation from source

# *The FEniCS challenge!*

- ❶ Install FEniCS on your laptop!

<http://fenicsproject.org/download/>

- ❷ Execute `demo_poisson.py`
- ❸ What are the main packages of the `dolfin` module?
- ❹ Which elements are supported in `dolfin`?
- ❺ Plot at least two finite elements from each row on page 4 and identify those elements you are most curious about!