**Seema Nanda**

**Numerical Analysis** **Assignment 2** **Jan 27, 2010 in class**

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Show all your work clearly. Please follow the instructions for assignments and homework as given in the course web page. Late homework will not be accepted. You may discuss problems with anyone but the work in the end should be your own. You MUST give credit to whoever helped you in your homework.

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**Root finding**

1. Compute a numerical approximation to the positive solution of $x^2 = 2$ by Newton's method. What function of $\epsilon_n$ and $\epsilon_{n+1}$ should you tabulate to observe the quadratic convergence? Do it.

2. Assume that Newton's method converges for a $f(x)$ which has a root of multiplicity $m$. Is convergence quadratic for Newton's method for $f(x)$? calculate the order of convergence. Suggest a change to Newton's method that will make convergence quadratic in this case.

3. Suppose you are using Newton's method to find the zeros of a cubic polynomial $f(x)$. Let the first guess $p_0$ lie halfway between any two of the three real distinct roots. In this case Newton's method converges to the third root in the first iteration. Check this and prove it.

4. Same equation, $x^2 = 2$, but this time, use the secant method. Choose $x_0$ and $x_1$ so the $\{x_n\}$ converge. Get your list of approximation errors $\epsilon_n$ , and find the exponent $s$ so $\frac{|\epsilon_n|}{|(\epsilon_{n+1})^s|} \rightarrow$ nonzero constant as $n \rightarrow \infty$.

5. Use Newton's method to find reciprocal of a number

   Consider the function $f(x) = a - \frac{1}{x}$

   Use Newton to estimate $\frac{1}{a}$.

6. Produce a table of $x$ *versus* $y$, where $y$ is defined implicitly as a function of $x$. Use $G(x, y) = 3x^7 + 2y^5 - x^3 + y^3 - 3$ and start at $x = 0$, proceeding in steps of 0.1 to $x = 10$.