Seema Nanda

Numerical Analysis **Assignment 4** **due: February 25, 2010 in class**

Please follow the instructions for assignments and homework as given in the course web page. You may discuss the problems and solutions with anyone but the work written up and submitted must be done on your own. Also programs must be written by you. (P) indicates programming is required

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1. **Norms and Conditioning of a matrix**

   (a) Using a 2-matrix norm, show that in the case of a symmetric matrix the condition number is the absolute value of the quotient of the largest and smallest eigenvalue of the matrix.

   (b) Prove that all $n \times n$ invertible matrices form a dense set in the set of all $n \times n$ matrices. That is show that if A is an $n \times n$ matrix then for $\epsilon > 0$ there is an invertible matrix B with $\|B - A\| < \epsilon$.

   (c) Prove that if $\|AB - I\| < 1$, then $2B - BAB$ is a better approximate inverse of A than B in the sense that $A(2B - BAB)$ is closer to I.

2. **(Iterative methods for solving $Ax = b$)**
   let $A \in R^{n\times n}$ be such that $A = (1 + \omega)P - (N + \omega P)$ with $P^{-1}N$ non singular and with real eigenvalues $1 > \lambda_1 \geq \lambda_2 \cdots \geq \lambda_n$. Find the values of $\omega \in R$ for which the following iterative method converges, $\forall x^{(0)}$, to the solution of the system $Ax = b$:

   $$(1 + \omega)Px^{(k+1)} = (N + \omega P)x^{(k)} + b, \quad k \geq 0 \tag{1}$$

   Can you determine the value of ω for which convergence is fastest.

3. Check that finding the roots of the zeros of a polynomial of deg $\leq n$ with real coefficients $P_n(x) = \sum_{k=0}^{n} a_k x^k, \ a_n \neq 0$
   is equivalent to determining the spectrum of the Frobenius matrix C in $R^{n\times n}$ associated with P_n.

   $$C = \begin{bmatrix} -(a_{n-1}/a_n) & -(a_{n-2}/a_n) & ... & -(a_0/a_n) \\ 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ . & . & ... & . \\ . & . & ... & . \\ 0 & 0 & ... & 1 \end{bmatrix}$$

   By Abel's theorem there exists in general no direct method to compute the eigenvalue of a matrix of size $n \geq 5$.

4. **Interpolation**

(a) What condition will have to be put on nodes x_0 and x_1 if the interpolation problem:
$p(x_i) = c_{i0}$, $p''(x_i) = c_{i2}$ for $i = 0, 1$
is to be solvable by a cubic polynomial for arbirtray c_{ij}.

(b) Draw a curve of your liking eg. a spiral or an oval on a sheet of graph paper. Select points in a rectangular distribution along the the curve and label points t_0, t_1, t_2,.... etc. Record the $x(t)$ and $y(t)$ values in a table. Then fit each of x and y to spline functions S annd S^*. Hence get $x = S(t)$ and $y = S^*(t)$ as parametric representations of the curve. Plot the resulting curve for two test cases using matlab. Submit your graph paper.