

Please follow the instructions for assignments and homework as given in the course web page. You may discuss the problems and solutions with anyone but the work written up and submitted must be done on your own. Also programs must be written by you. (P) indicates programming is required

1. **Lagrange form of interpolating polynomial:** Show that the Lagrange form of the interpolating polynomial approximation of  $f(x)$  can be written as:

$$P_n(x) = f[x_0] + \sum_{k=1}^n f[x_0, x_1, \dots, x_k] \prod_{i=0}^{k-1} (x - x_i) \quad (1)$$

and the approximation error is  $\frac{f^{n+1}(\psi_x)}{(n+1)!} \prod_{i=0}^{k-1} (x - x_i) = f[x_0, x_1, \dots, x_k, x] \prod_{i=0}^{k-1} (x - x_i)$ .

2. **Composite Simpson's rule** The equation  $\int_0^x \frac{1}{\sqrt{(2\pi)}} e^{-t^2/2} dt = 0.45$  can be solved for  $x$  by using Newton's method with  $f(x) = \int_0^x \frac{1}{\sqrt{(2\pi)}} e^{-t^2/2} dt - 0.45$  and  $f'(x) = \frac{1}{\sqrt{(2\pi)}} e^{-x^2/2}$ . To evaluate  $f$  at the approximate  $p_k$ , we need a quadrature formula to approximate  $\int_0^{p_k} \frac{1}{\sqrt{(2\pi)}} e^{-t^2/2} dt$ . Find a solution to  $f(x) = 0$  accurate to within  $10^{-5}$  using Newton's method with  $p_0 = 0.5$  and the composite Simpson's rule.
3. Use **Richardson Extrapolation** to calculate  $\ln(a)$  by differentiation of  $f(x) = a^x$  for  $x > 0$ .
4. **Gaussian Quadrature:** Let  $p_m(x)$  be a set of polynomials orthogonal over  $(a, b)$  with respect to the weight  $w(x) > 0$ . That is,  $p_m(x)$  is an  $m$ th degree polynomial and  $\int_a^b p_m(x)p_n(x)w(x)dx = 0$  unless  $m = n$ . Show that all the zeros of the  $p_m(x)$  are real, have multiplicity 1, and lie in  $(a, b)$ .
5. What happens in the theory of Gaussian Quadrature if the function  $w(x)$  in  $\int_a^b f(x)w(x)dx$  is not positive.
6. Prove that if an  $m$ th degree polynomial  $f$  is correctly integrated by the formula

$$\int_0^1 f(x)dx = \sum_{i=0}^n Af(i) \quad (2)$$

then the same is true for the formula

$$\int_{t_0}^{t_0+h} f(x)dx = h \sum_{i=0}^n Af(t_0 + ih) \quad (3)$$