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Numerical Analysis **Assignment 6** **due: April 6, 2011 in class**

Please follow the instructions for assignments and homework as given in the course web page. You may discuss the problems and solutions with anyone but the work written up and submitted must be done on your own. Also programs must be written by you.

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1. **Consistency:** Consider the m step multistep method: $y_0 = \alpha_0, \ldots, y_m = \alpha_m$ $y_{i+1} = a_m y_i + a_{m-1} y_{i-1} + \cdots + a_0 y_{i-(m-1)} + h \sum_{k=0}^{m} b_{m-k} f(t_{i-(k-1)}, y_{i-(k-1)})$ where $a_0, \ldots a_m$ and $b_0, \ldots, b_m$ are constants.

   A multistep method is consistent if $\lim_{h \downarrow 0} \alpha_i = y_i$ for $i = 1, ..., m$ and if

   $$\tau_{i+1}(h) = \frac{1}{h}[y_{i+1} - \sum_{k=0}^{m} a_{m-k} y_{i-k} + h \sum_{k=0}^{m} b_{m-k} f(t_{i-(k-1)}, y_{i-(k-1)})] \tag{1}$$

   goes to 0 as $h \downarrow 0$ i.e.

   $$\lim_{h \downarrow 0} max_{i>m} |\tau_i(h)| = 0 \tag{2}$$

   For multistep methods, there is an easy check for consistency: (2) holds if and only if $\sum_{k=0}^{m} a_k = 1$ and $-\sum_{k=0}^{m} k a_k + \sum_{k=0}^{m} b_k = 1$. Prove this.

2. **Analysis of multistep methods:**

   Investigate the convergence of the following multistep methods by analysing the consistency and the stability. If convergent determine the order of convergence.

   (a) $y_{j+1} - y_{j-1} = 2hf(x_j, y_j)$ (explicit mid-point rule)

   (b) $y_{j+1} - y_{j-1} = hf(x_{j+1}, y_{j+1})$

   (c) $\frac{3}{2} y_{j+1} - 2y_j + \frac{1}{2} y_{j-1} = hf(x_{j+1}, y_{j+1})$

3. **Fibonacci series:** The famous Fibonacci series is given by the recursion $u_0 = 0, u_1 = 1, u_{n+1} = u_n + u_{n-1}$ for $n = 1, 2, 3, \ldots$

   (a) Determine a fundamental system of this linear difference equation. Use it to derive a formula for the elements in the Fibonacci series.

   (b) Does this linear difference equation satisfy the root condition?

   (c) Consider the perturbed initial values $\tilde{u}_0 = 1 + \epsilon_1$ and $\tilde{u}_1 = 1 + \epsilon_2$. Prove via induction that the resulting series satisfies $u_n = u_n(1+\epsilon_2) + u_n \epsilon_1$ for $n = 2, 3, 4, \ldots$ What do you conclude?

4. **Region of stability**

   Determine the region of stability for the methods in problem 2 above.