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Please follow the instructions for assignments and homework as given in the course web page. You may discuss the problems and solutions with anyone but the work written up and submitted must be done on your own. Also programs must be written by you.

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### 1. Boundary Value Problems:

- (a) What is the first variational equation for :

$$y'' = \cos(ty) + \sin(t^2y') \quad (1)$$

Can the first variational equation be solved numerically by itself, or must it be solved simultaneously with equation for  $y''$ ?

- (b) What is the first variational equation for the following linear differential equation:

$$x'' = a(t) + b(t)x + c(t)x' \quad (2)$$

- (c) Solve the 2 point boundary value problem

$$y'' + y' + y = -(x^2 + x + 1) \quad (3)$$

with  $y(0) = 0$  and  $y(1) = 0$ , using the shooting method and your favorite method for an initial value problem. Check your numerical results by finding the exact solution.

- (d) Try to solve the following problem using the same method as in previous problem.

$$u'' - 100u = 0 \quad (4)$$

with  $u(0) = 0$  and  $u(1) = 0$ . The exact solution is given by

$$u(t) = \frac{1}{1 - e^{-20}} e^{-10t} - \frac{e^{-20}}{1 - e^{-20}} e^{10t}. \quad (5)$$

Discuss the discrepancies between your best possible solution and the exact solution. What difficulties did you encounter, if any, in obtaining your numerical solution?