AERODYNAMIC OPTIMIZATION OF WINGS FOR PROPELLER–DRIVEN AIRCRAFT

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ABSTRACT

Aerodynamic optimization of wings for aircraft driven by propellers or similar rotors is carried out here using an optimization algorithm PROWING developed by the authors. The algorithm includes an extended lifting line theory to account for propeller slipstream effects and an optimization routine using the weights of Bezier polynomials as control parameters. Novel wing designs can be generated using PROWING, and yield lower induced and total drag subject to aerodynamic constraints such as given total lift and geometric constraints such as given root chord, tip chord, wing area and bounds on wing twist.

INTRODUCTION

Since the early 1990's there has been a decline in the usage of turboprops because of cheap oil. At that time jets were preferred as they were affordable, faster and quieter. However owing to climate change concerns and connectivity demands turboprops are now once again becoming more attractive. Hence, there is a need to explore new technologies associated with turboprops and optimize them for better performance.

Pursuing this objective a few important aspects of wing-propeller interaction are discussed in the following paragraphs.

For aircraft powered by propellers or other rotors, one of the major concerns is the interaction between the propeller slipstream and the wing (for significant earlier work see [1, 2, 3]). Part of the wing immersed in the propeller slipstream will experience an increase in the axial velocity and a change in the downwash field. Due to the increase in the dynamic pressure the local lift force increases and, as a consequence, there is an increase in the induced drag as well, because of non-optimal lift distribution. This increase in the induced drag can be minimized by appropriately shaping the planform or incorporating twist distribution or by doing both (Kroo[2], Veldhuis[3]). Therefore, in order to investigate the interaction between the propeller slipstream and the wing, an optimization program called PROWING was developed. It is based on an extended lifting line theory and an optimization method which uses the weights of Bezier polynomials as control parameters to generate novel wing designs can be generated.

PROBLEM FORMULATION

Figure 1 shows a schematic of a wing in a tractor-propeller configuration. The wing has a semispan of s and a chord distribution $c(y)$ where y is the spanwise coordinate. Part of the wing immersed in the propeller slipstream experiences an increase in the axial velocity $V(y)$ and an additional downwash $w_p(y)$ along with the downwash due to the trailing vortices of the wing $w_w(y)$.

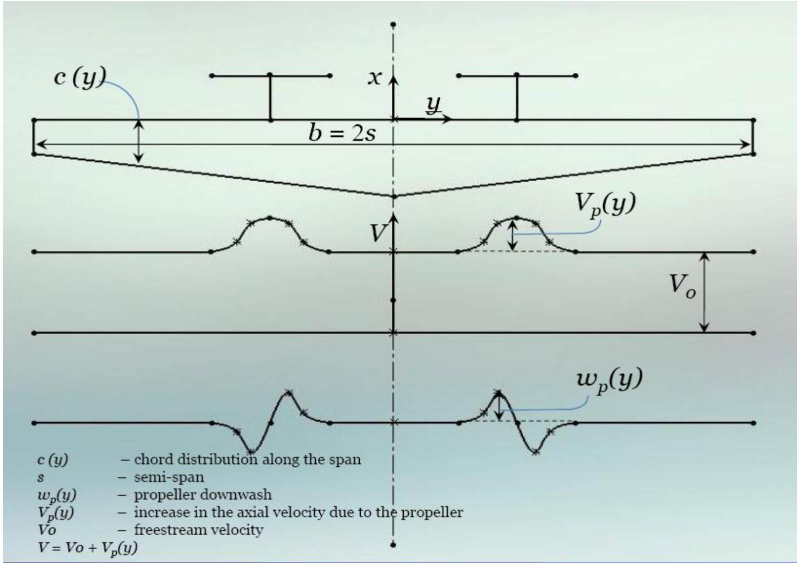
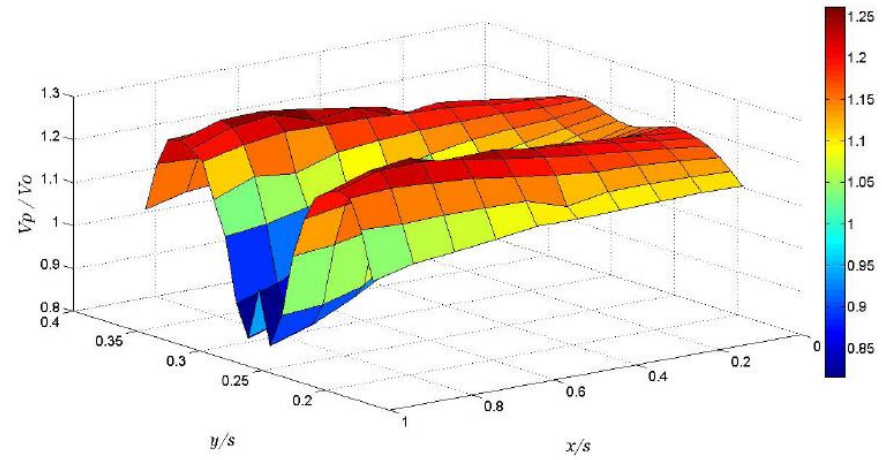
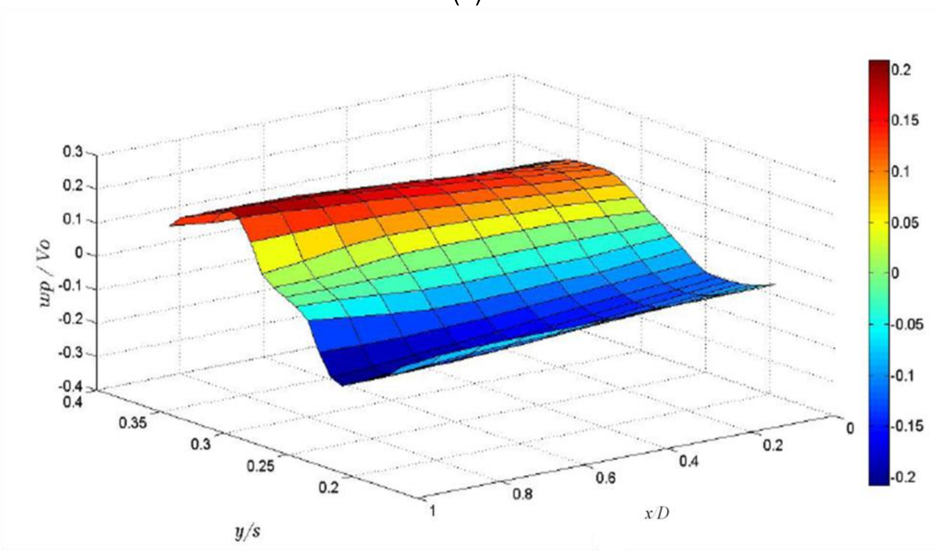


Figure 1: *Schematic representation of a wing in tractor-propeller configuration*



(a)



(b)

Figure 2: *(a) plot of time averaged axial velocity, (b) plot of time averaged downwash field*
Source: *Flow field computed at NAL, Bangalore* [4]

Figure 2 shows the time averaged variation of axial velocity and downwash in the propeller slipstream. The propeller is located at x/D = 1.0 where D denotes the diameter of the propeller. y/s runs along the span of the wing across the diameter of the propeller. In order to account for the propeller slipstream effects, an extended lifting line theory was developed. The mathematical formulation is as follows:

Given:

$$V(y) = V_0 + V_p(y) \quad (1)$$
$$w_p(y) \quad (2)$$

where $V_p(y)$ and $w_p(y)$ are the time-averaged axial velocity and downwash respectively and V_0 is the free stream velocity.

The total downwash $w(y)$ is the sum of the downwash due to the wing and the propeller. If $\Gamma(y')$ is the circulation at y' and y is the location at which the total downwash is needed, we have

$$w(y) = w_w(y) + w_p(y) \quad (3)$$
$$= \frac{1}{4\pi}\int_{-s}^{+s} \frac{1}{y-y'}\frac{d\Gamma}{dy'}dy' + w_p(y) \quad (4)$$

The effective angle of attack which the aerofoil section sees is given by

$$\alpha_e(y) = \alpha(y) - \frac{w_w(y) + w_p(y)}{V(y)} \quad (5)$$

where $\alpha(y) = \alpha_g - \alpha_t(y)$, α_g is the geometric angle of attack and $\alpha_t(y)$ is the twist along the span measured from the angle of zero lift as indicated in figure 3.

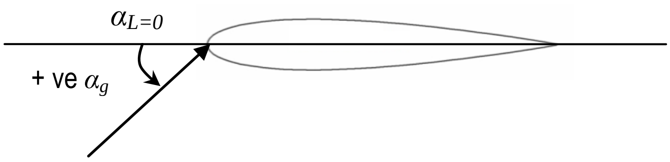


Figure 3: *Picture of aerofoil showing counter clockwise measurements are positive from angle of zero lift*

The lift and induced drag are given respectively by

$$L = \int_{-s}^{+s} \rho V(y)\Gamma(y)dy \quad (6)$$

$$D_i = \int_{-s}^{+s} \rho w(y)\Gamma(y)dy \quad (7)$$

Transforming the coordinate system from y to θ by putting $y = -s\cos\theta$ and expanding Γ in Fourier series [4] with A_k as the Fourier coefficients, we have

$$\Gamma = 4sV_o\sum_{k=1}^{\infty} A_k \sin k\theta \quad (8)$$

The fundamental equations to determine downwash, the Fourier coefficients, lift and induced drag will respectively transform into the following set:

$$w(y)=v_o\frac{\sum_{k=1}^{\infty} kA_k \sin k\theta}{\sin\theta} + w_p(y) \tag{9}$$

$$\left(\sum_{k=1}^{\infty} A_k \sin k\theta\right)(k\mu + \sin\theta) = \mu\frac{V(\theta)}{V_o}\sin\theta\left(\alpha - \frac{w_p(\theta)}{V(\theta)}\right); \quad \mu = \frac{a_o c}{8s} \tag{10}$$

$$L \;=\; 4\rho V_o s^2 \int_0^{\pi} V(\theta)\sin\theta \sum_{k=1}^{\infty} A_k \sin k\theta d\theta \tag{11}$$

$$D_i \;=\; 2\pi s^2 \rho V_o^2 \sum_{k=1}^{\infty} kA_k^2 + A_k B_k \tag{12}$$

where

$$B_k \;=\; \frac{2}{\pi}\int_0^{\pi} \frac{w_p(y)}{V_o}\sin k\theta \sin\theta \tag{13}$$

NUMERICS

We solve the lifting line equation to determine the Fourier coefficients by the method of collocation using a least squares approach.

Method of solution

Let n_c and $n_f(< n_c)$ be the number of collocation points along the span and number of Fourier coefficients respectively. For validation n_c = 16, n_f = 8, For wing-propeller configuration n_c = 320, n_f = 48. If we denote LL = 0 as the lifting line equation then the least squares formulation will be as follows:

$$\mathrm{LL}\ (\theta, A_1 \ldots A_{nf}) = (\textstyle\sum_{k=1}^{n_f} A_k \sin k\theta)\ (k\mu + \sin\theta) - \ \mu\ \left(\frac{V(\theta)}{V_o}\right)\sin\theta\ \left(\alpha - \frac{w_p}{V(\theta)}\right) = 0 \tag{14}$$

$$\frac{\partial}{\partial A_j}\textstyle\sum_{i=1}^{n_c}\left[\mathrm{LL}\left(\theta_i, A_1 \ldots A_{nf}\right)\right]^2 = 0; \quad j = 1 \ldots . n_f \tag{15}$$

Validation

The code was validated against Glauert's [4] results on monoplane coefficients varying with taper ratio for a wing alone case.

In order to determine the number of Fourier modes and collocation points for wing–propeller configuration, a convergence study was conducted. From figure 4 it can be seen that 48 Fourier modes and 320 collocation points are sufficient to determine the circulation distribution along the span for a reference wing with a taper of 0.45 wing and a 3° linear washout with a propeller as a propulsive unit. Figure 5 shows the circulation distribution for the reference wing–propeller configuration.

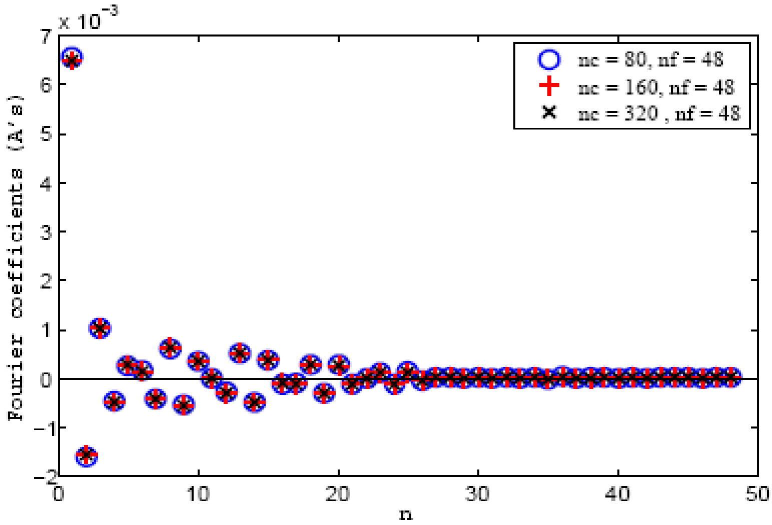


Figure 4: *Plot of variation of fourier modes with collocation points*

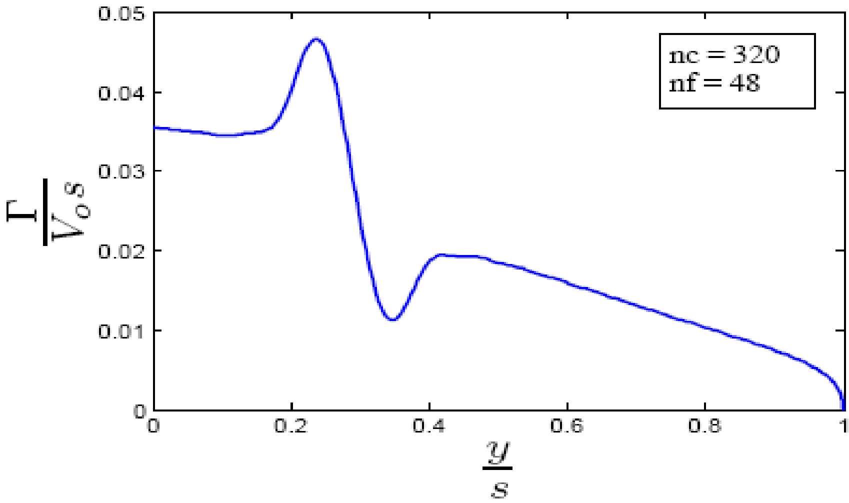
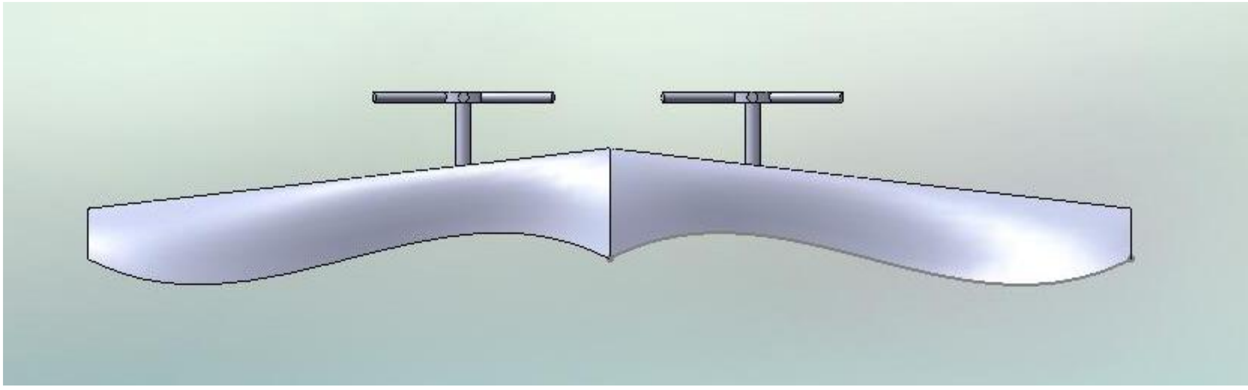


Figure 5: *Plot of circulation for a reference wing – propeller configuration*

RESULTS

Figure 6 shows an example of the optimized wing design obtained in this work for a wing with for a wing with a straight leading edge, a taper of 0.45 and a 3° linear washout. The constraints included C_L= 0.27, wing area, root chord, tip chord and bounds on twist $(-14^o \leq \alpha_t \leq 14^o)$. The optimized wing gives an induced drag reduction of 19.3 % and a total drag reduction of 8.74 %.



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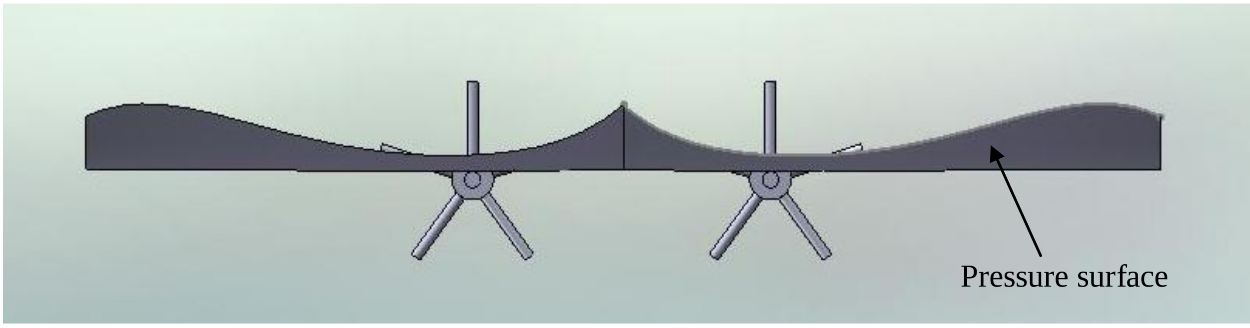

(b)

Figure 6: *(a) Planform view (chord distribution)*
(b) rear view (looking from the trailing edge), the twist distribution is magnified by 20 times for better visualization of the twist!!!

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