TA2 Test Case

Praveen. C[1] R. Duvigneau[2]

[1] Tata Institute of Fundamental Research
Center for Applicable Mathematics
Bangalore 560065

[2] Projet Opale, INRIA Sophia Antipolis

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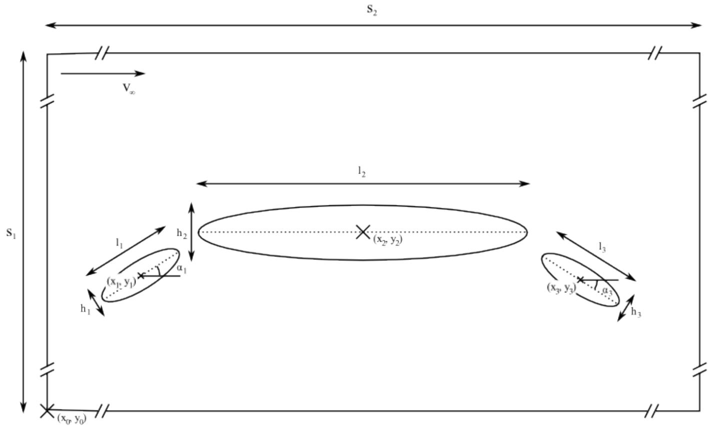
TA2 test case

Aerodynamic reconstruction problem

Recovery of the original position of two ellipses using Navier-Stokes flows for Re = 100 and Re = 500



TA2 test case: Design variables, bounds, target

Design parameters:

$$
\begin{aligned}
-10.0 &\le x_1 \le -6.5 \\
-1.5 &\le y_1 \le 0.0
\end{aligned}
$$

position of the ellipse 1

$$-10.0^\circ \le \alpha_1 \le 0.0^\circ$$

clockwise angle of the ellipse 1

$$
\begin{aligned}
7.25 &\le x_3 \le 10.0 \\
-1.5 &\le y_3 \le 0.0
\end{aligned}
$$

position of the ellipse 3

$$0.0^\circ \le \alpha_3 \le 10.0^\circ$$

clockwise angle of the ellipse 3

In addition, the ellipses/ellipsoids must not be overlapping.

Target:

$$\{x_1, y_1, \alpha_1, x_3, y_3, \alpha_3\} = \{-7.0, -0.5, -3.0^o, 7.5, -0.5, 3.0^o\}$$

TA2 test case: Flow conditions

- Incompressible fluid (Navier-Stokes laminar flow)
  Our results are for $M_\infty = 0.2$
- Reynolds number $Re = 100$ or 500
  Our results are for $Re = 500$ and 1000
- Angle of attack $\alpha = 5$ deg.

Recovery of position by minimizing the pressure difference

$$\min \ f = \int_{\Gamma_1} (p_1 - p_1^*)^2 + \int_{\Gamma_2} (p_2 - p_2^*)^2 + \int_{\Gamma_3} (p_3 - p_3^*)^2$$

Flow solver: flo2d

- Finite volume scheme
- Unstructured, triangular grids
- Roe flux
- MUSCL reconstruction
- Implicit scheme

Source code of flo2d available online
`http://flo2d.googlecode.com`

Grid for CFD

- x^0 = Design variables corresponding to middle of design space
- G_0 = Grid corresponding to x_0 (Reference grid)
- To obtain grid for any other configuration, we deform the reference grid using Radial Basis Function interpolation.
- Grid points on middle ellipse and outer boundary are fixed
- Grid used in this work
  - 33438 vertices
  - 65994 triangles
- Grid generated using delaundo

Grid deformation

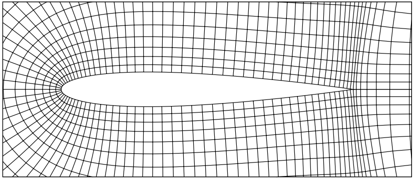
- Interpolate displacement of surface points to interior points using RBF

$$\tilde{f}(x,y) = a_0 + a_1 x + a_2 y + \sum_{j=1}^{N} b_j |\vec{r} - \vec{r}_j|^2 \log |\vec{r} - \vec{r}_j|$$

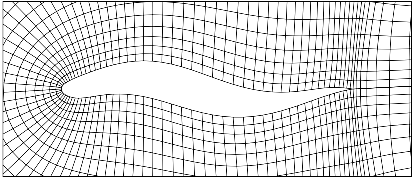
where $\vec{r} = (x, y)$

- Results in smooth grids

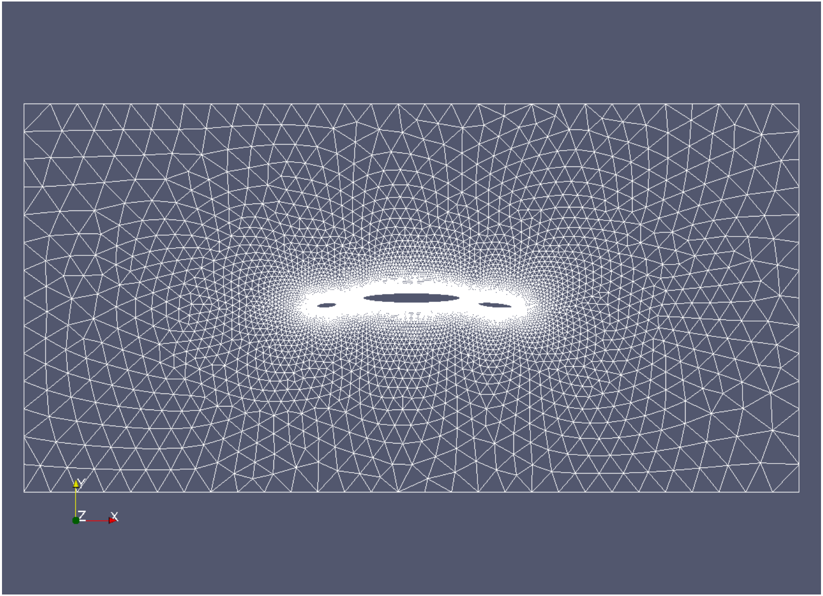
Initial grid



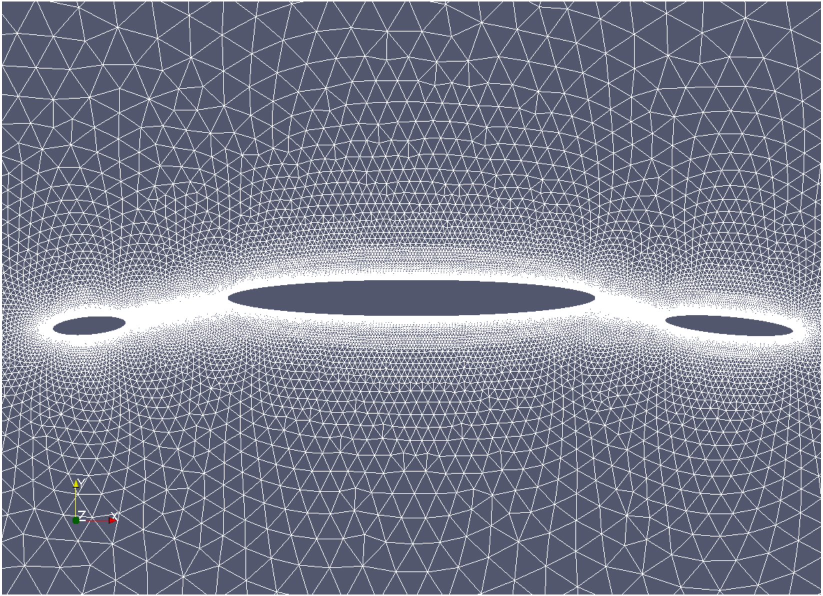
Deformed grid



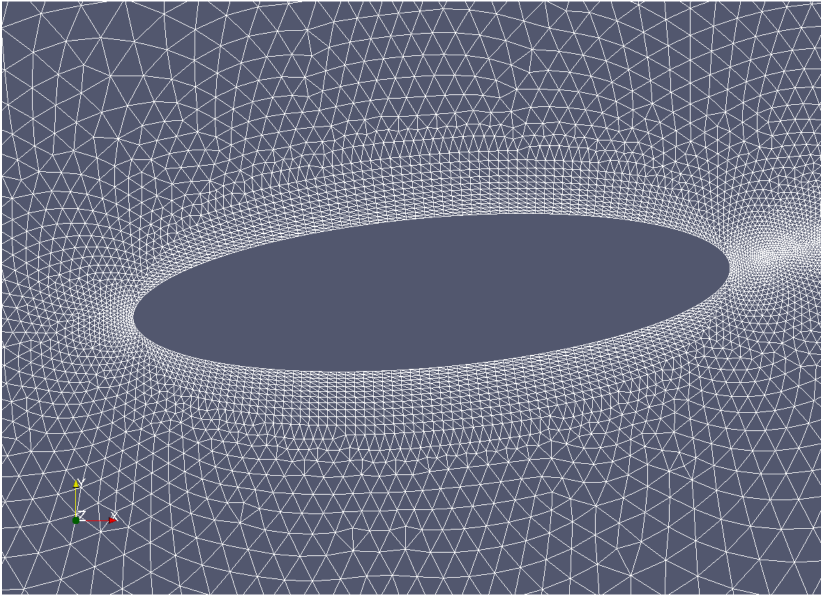
Reference grid



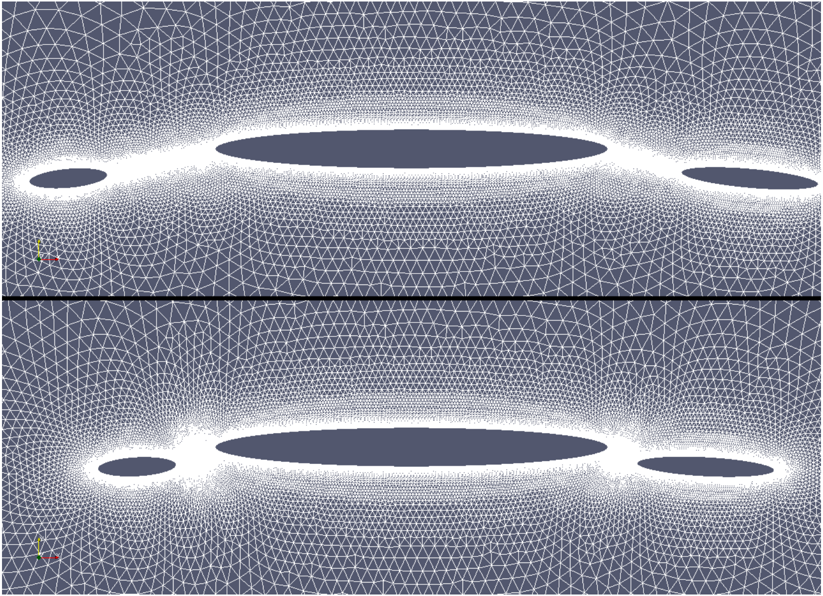
Reference grid



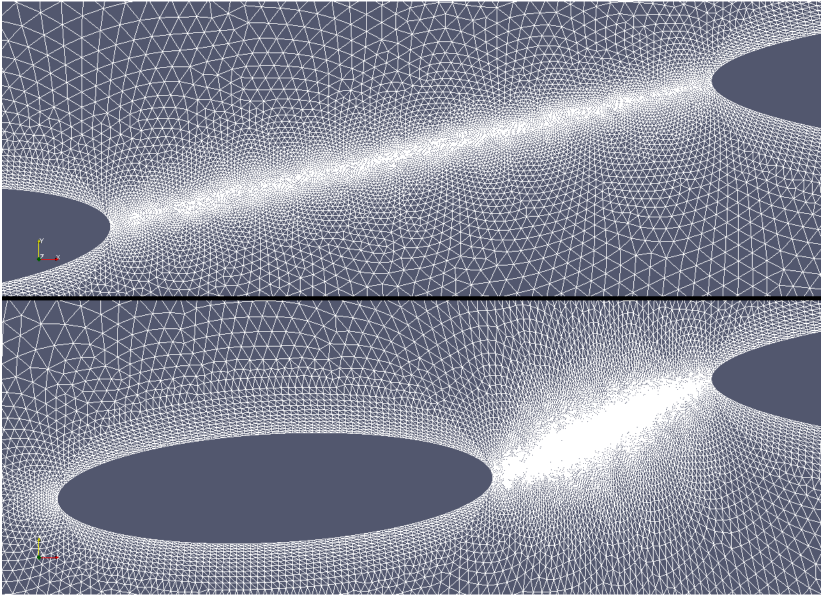
Reference grid: Around first ellipse



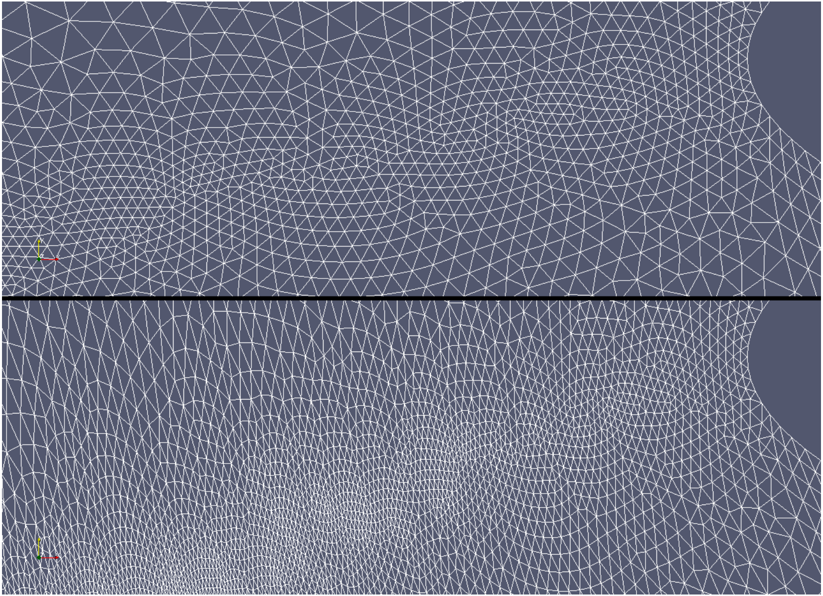
Reference and target grid



Reference and target grid: B/w first and second ellipse



Reference and target grid: B/W first and second ellipse

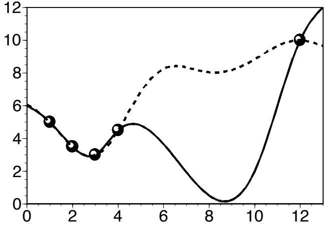


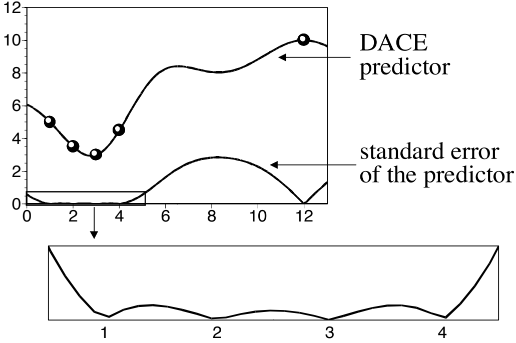
Global metamodel-based optimization

- Global models: provide global trends in objective function
  - Faster convergence towards global optimum
- Metamodels are approximate, inaccurate
- Not possible to construct accurate metamodel in one-shot
- Difficult to construct uniformly accurate model in high dimensions
  - *Curse of dimensionality*
- Model must be accurate in regions of optima
- But need to sufficiently explore the design space
- Balance between exploration and exploitation

Gaussian process models

- Treat results of a computer code as a stochastic process !!!
- Provides an estimate of the variance in predicted value





Merit functions

- Statistical lower bound

$$f_M(x) = \tilde{J}(x) - \kappa \tilde{s}(x)$$

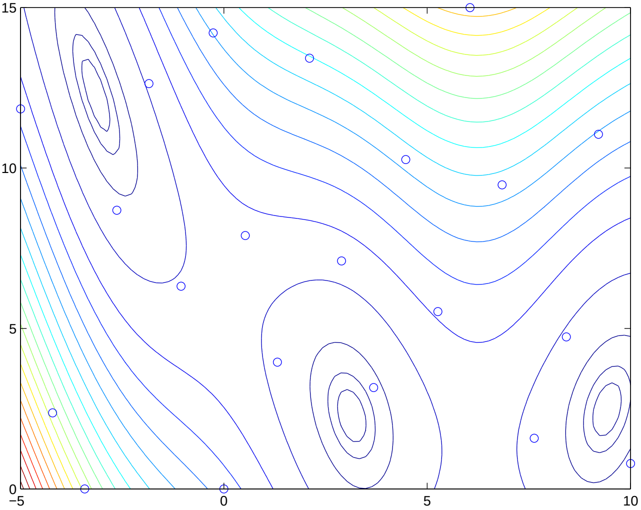
- Probability of improvement

$$\mathrm{PoI}(x) = \Phi\left(\frac{T - \tilde{J}(x)}{\tilde{s}(x)}\right)$$

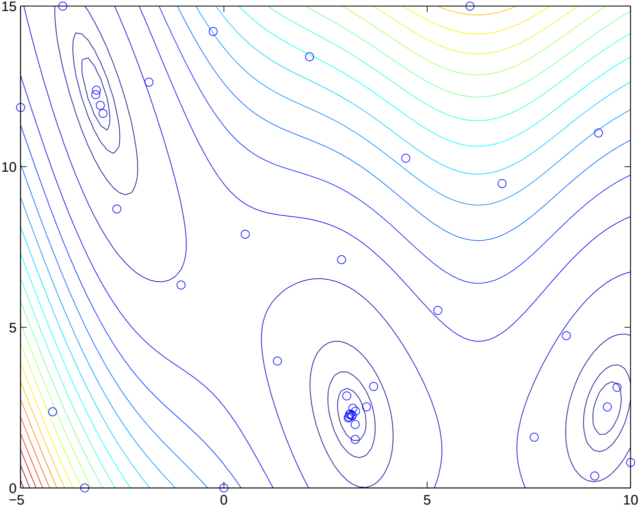
- Expected improvement

$$\mathrm{EI}(x) = \tilde{s}(x)[u\Phi(u) + \phi(u)], \quad u(x) = \frac{J_{\min} - \tilde{J}(x)}{\tilde{s}(x)}$$

Minimization of 2-D Branin function: Initial database



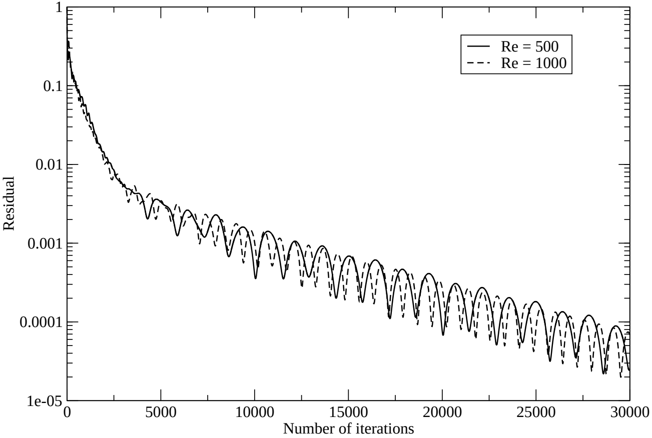
Minimization of 2-D Branin function: after 20 iter



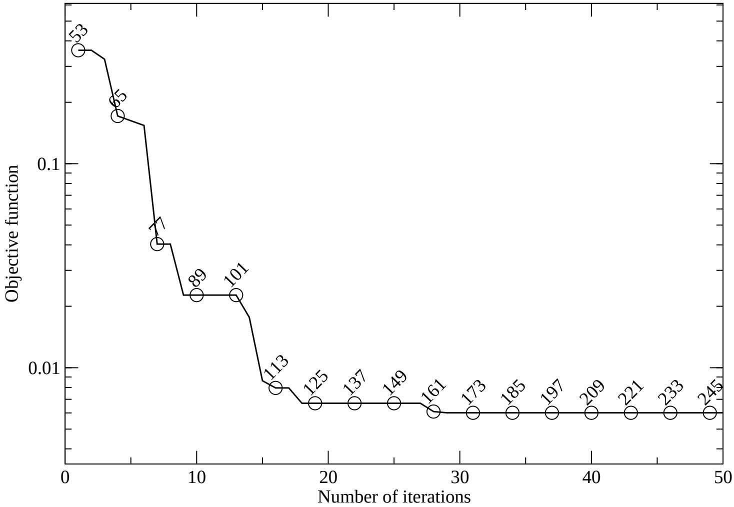
Optimization test

- 6 design variables
- Initial database of 48 using LHS
- 4 merit functions based on statistical lower bound with $\kappa = 0, 1, 2, 3$
- Gaussian process models
- Merit functions minimized using PSO

Convergence of CFD iterations for target configuration

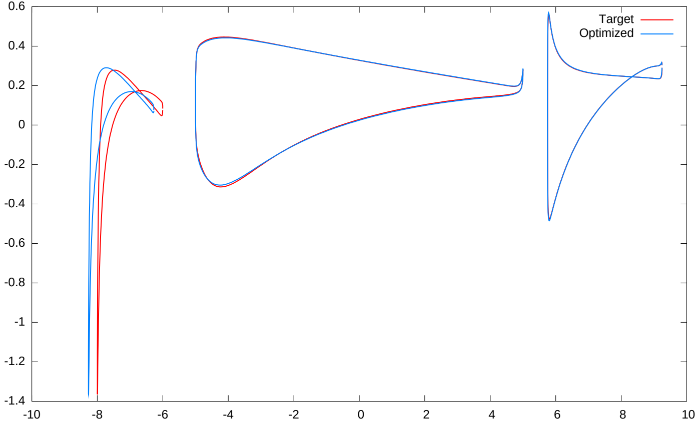


Convergence of optimization for Re=500



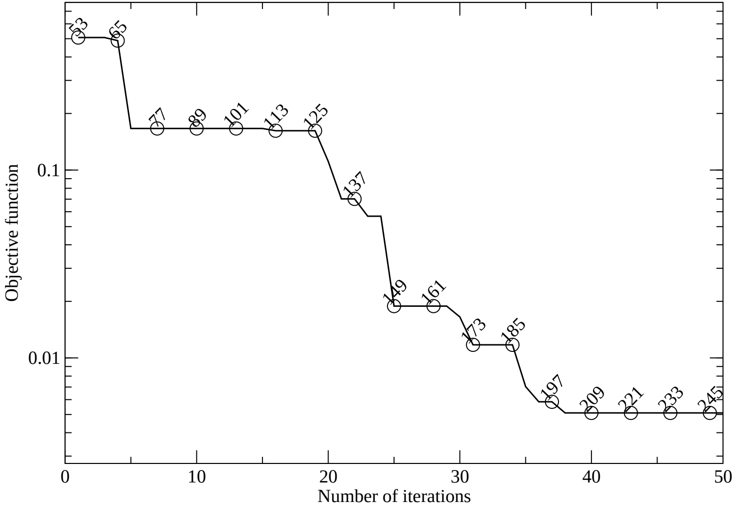
Best objective function value $= 6.00 \times 10^{-3}$ (normalized) or 1.25×10^{-4}

Pressure coefficient for $Re = 500$



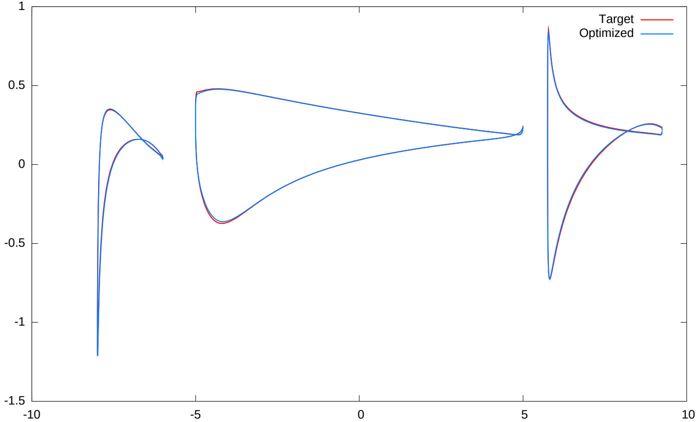
	x_1	y_1	α_1	x_3	y_3	α_3
Target	-7.5	-0.5	-3.5	7.5	-0.5	3.0
Opt	-7.271	-0.541	-3.232	7.494	-0.518	3.120

Convergence of optimization for Re=1000



Best objective function value = 5.09×10^{-3} (normalized) or 1.06×10^{-4}

Pressure coefficient for $Re = 1000$



	x_1	y_1	α_1	x_3	y_3	α_3
Target	-7.5	-0.5	-3.5	7.5	-0.5	3.0
Opt	-6.993	-0.504	-2.675	7.498	-0.497	2.641

Summary

- Grid is deformed in smooth way by RBF interpolation.
  We expect objective function to depend continuously on the design variables.
- CFD has good convergence and pressure is smooth on the ellipses
- Objective is reduced by 3 orders of magnitude for both Reynolds numbers
- But for Re=500, position of first ellipse is not well recovered
- Objective function could be insensitive to position of first ellipse.
  This behaviour has been seen by other presentations in the first workshop.
- For Re=1000, position is recovered well but both angles are far off from the target values.
  But pressure looks quite close to target pressure.
- Global optimization methods not able to precisely locate the optimum. Performance could be improved by a trust region approach and/or using some gradient information.