

TA5 Test Case

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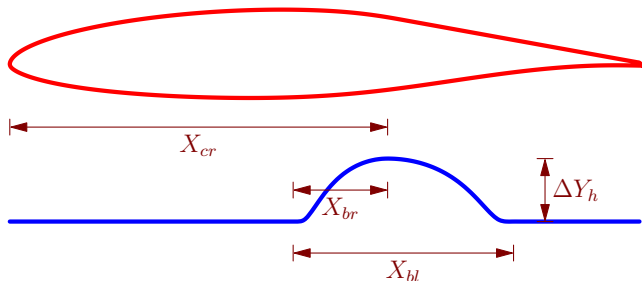
²Projet Opale, INRIA Sophia Antipolis

Integrated Multiphysics Simulation & Design Optimization
Database Workshop for multiphysics optimization software validation
Presentation of the Academic Test Case Results
Agora, Jyväskylä, Finland
December 3-4, 2009

- Optimize RAE5243 airfoil to reduce drag under lift constraint

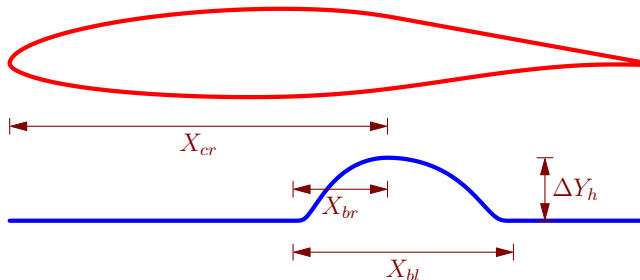
Mach	Re	C_l	Flow condition
0.68	19 million	0.82	Fully turbulent

- Modify shape of upper airfoil surface by adding a bump



Airfoil chord is taken to be unity

Bound constraints for bump parameters



$$0 < X_{cr} < 1$$

$$0 < X_{br} < X_{bl}$$

$$0 < X_{bl} < 0.4$$

$$0 < \Delta Y_h < 0.05$$

Modification of bound constraints

- If X_{br} and/or $X_{bl} - X_{br}$ is too small, then CFD grid will not be able to resolve the bump.

$$X_{br} > L_{\min}, \quad X_{bl} - X_{br} > L_{\min}$$

- Also, we restrict X_{cr} and ΔY_h

$$\begin{aligned} 0.4 &< X_{cr} < 0.8 \\ L_{\min} &< X_{br} < X_{bl} \\ 2L_{\min} &< X_{bl} < 0.4 \\ 0 &< \Delta Y_h < 0.01 \end{aligned}$$

In the computations, we use $L_{\min} = 0.05$

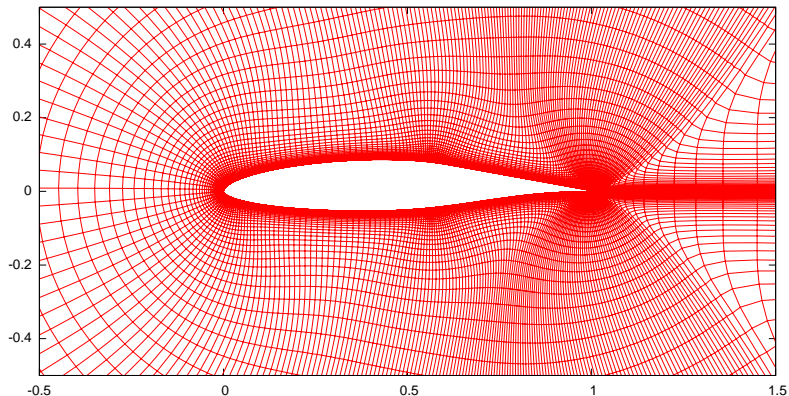
Based on the ISAAC code of Joseph Morrison
<http://isaac-cfd.sourceforge.net>

- Finite volume scheme
- Structured, multi-block grids
- Roe flux
- MUSCL reconstruction
- Implicit scheme, grid sequencing, multigrid
- Wilcox $k - \omega$ turbulence model

Source code of NUWTUN available online
<http://nuwtun.berlios.de>

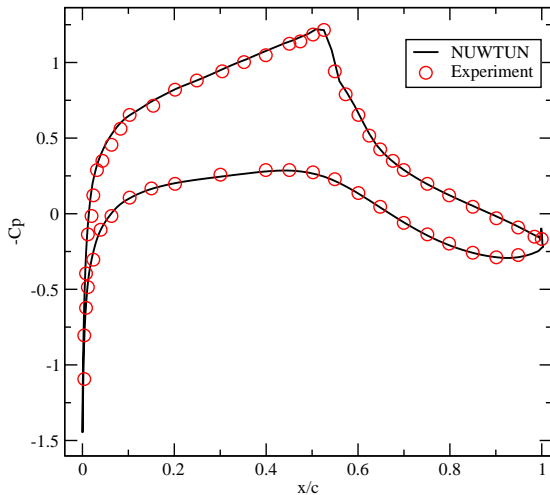
Grid for CFD

C-grid of size 353×97 , 270 points on airfoil,
 $y^+ < 1.5$, outer boundary at 20 chords



Validation with experiments

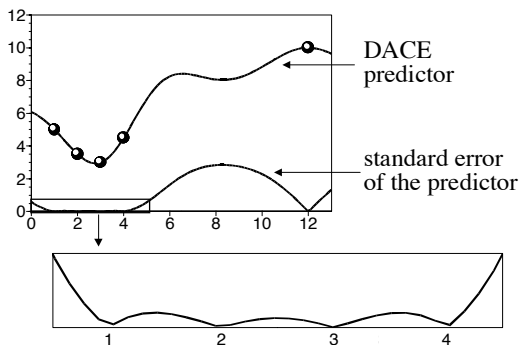
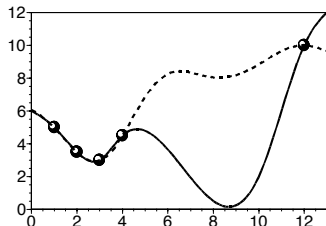
$M = 0.68$, $Re = 19$ million, $\alpha = 0.77$ deg.



- Global models: provide **global trends** in objective function
 - ▶ Faster convergence towards global optimum
- Metamodels are approximate, inaccurate
- **Not possible** to construct accurate metamodel **in one-shot**
- Difficult to construct **uniformly accurate** model in high dimensions
 - ▶ *Curse of dimensionality*
- Model must be **accurate** in **regions of optima**
- But need to sufficiently **explore** the design space
- Balance between **exploration** and **exploitation**

Gaussian process models

- Treat results of a computer code as a **stochastic** process !!!
- Provides an estimate of the **variance** in predicted value



- Statistical lower bound

$$f_M(x) = \tilde{J}(x) - \kappa \tilde{s}(x)$$

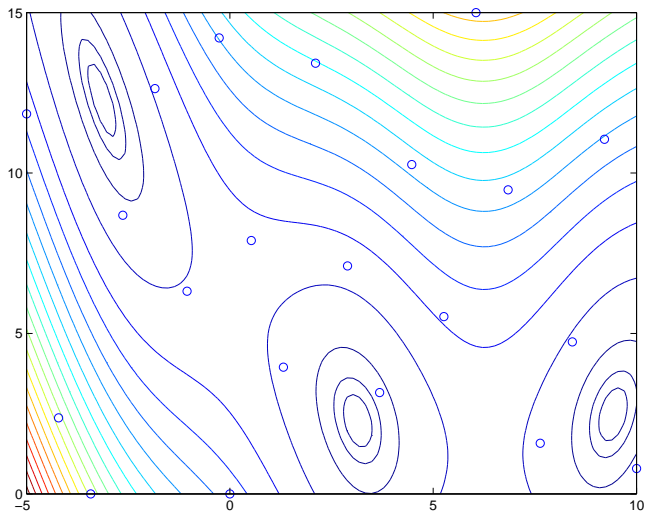
- Probability of improvement

$$\text{PoI}(x) = \Phi \left(\frac{T - \tilde{J}(x)}{\tilde{s}(x)} \right)$$

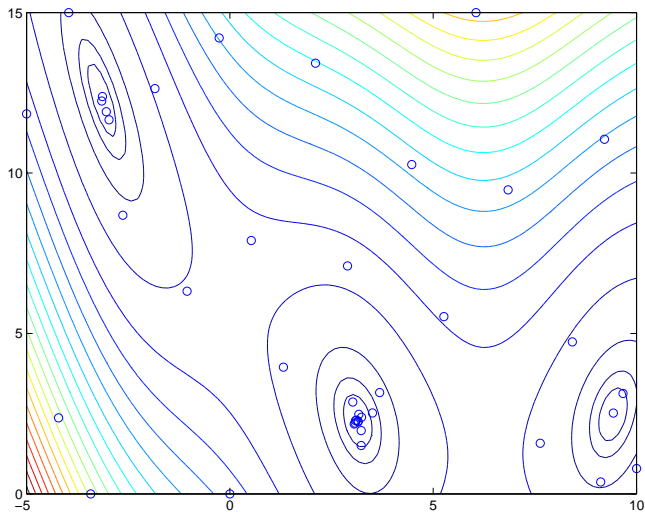
- Expected improvement

$$\text{EI}(x) = \tilde{s}(x)[u\Phi(u) + \phi(u)], \quad u(x) = \frac{J_{\min} - \tilde{J}(x)}{\tilde{s}(x)}$$

Minimization of 2-D Branin function: Initial database



Minimization of 2-D Branin function: after 20 iter



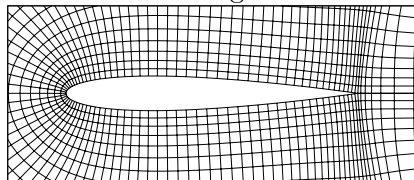
- **Interpolate** displacement of **surface** points to **interior** points using RBF

$$\tilde{f}(x, y) = a_0 + a_1x + a_2y + \sum_{j=1}^N b_j |\vec{r} - \vec{r}_j|^2 \log |\vec{r} - \vec{r}_j|$$

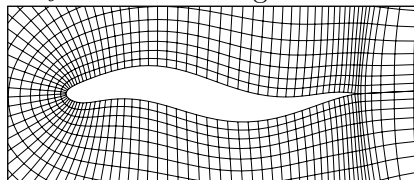
where $\vec{r} = (x, y)$

- Results in **smooth** grids

Initial grid



Deformed grid



Reference solution and design variables

Result	α deg.	C_l	C_d	C_{d_p}	C_{d_v}
Present	2.5	0.8244	0.01627	0.01052	0.005757
Qin et al.	-	0.82	0.01622	0.01063	0.005586

α range for optimization: $2 < \alpha < 3$

$$0.4 < X_{cr} < 0.8$$

$$L_{\min} < X_{br} < X_{bl}$$

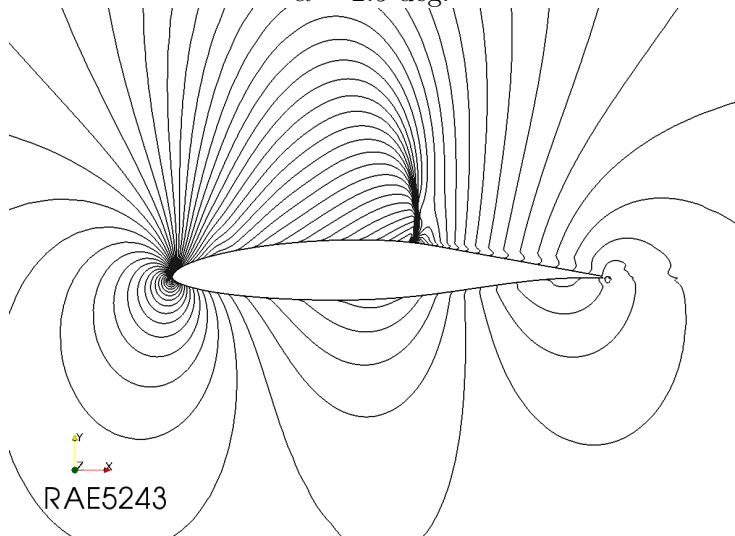
$$2L_{\min} < X_{bl} < 0.4$$

$$0 < \Delta Y_h < 0.01$$

$$2 < \alpha < 3$$

Reference solution: Pressure

$\alpha = 2.5$ deg.



Transformed design variables

- Transformations

$$x_1 = X_{cr}$$

$$x_2 = \frac{X_{br} - L_{\min}}{X_{bl} - 2L_{\min}}$$

$$x_3 = X_{bl} - 2L_{\min}$$

$$x_4 = \Delta Y_h$$

$$x_5 = \alpha - 2.5$$

$$X_{cr} = x_1$$

$$X_{br} = x_2 x_3 + L_{\min}$$

$$X_{bl} = x_3 + 2L_{\min}$$

$$\Delta Y_h = x_4$$

$$\alpha = x_5 + 2.5$$

- Bounds

$$0.4 < x_1 < 0.8$$

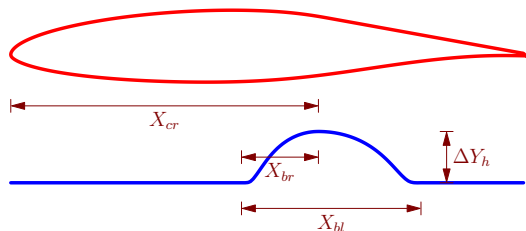
$$0 < x_2 < 1.0$$

$$0 < x_3 < 0.4 - 2L_{\min}$$

$$0 < x_4 < 0.01$$

$$-0.5 < x_5 < 0.5$$

Bump function



$y_l^{(0)}, y_u^{(0)}$: lower and upper curves of RAE5243 airfoil

$$y_u(x) = y_u^{(0)}(x) + y_b(x)$$

$$y_l(x) = y_l^{(0)}(x)$$

Bump function

$$y_b(x) = \begin{cases} 0 & x \leq X_{cr} - r_1 \text{ or } x \geq X_{cr} + r_2 \\ \Delta Y_h \cdot \text{Cubic}(x) & X_{cr} - r_1 < x \leq X_{cr} \\ \Delta Y_h \cdot \text{Cubic}(x) & X_{cr} < x < X_{cr} + r_2 \end{cases}$$

$$r_1 = X_{br}, \quad r_2 = X_{bl} - X_{br}$$

Constrained problem

$$\min C_d \text{ subject to } C_l = C_{l_0}$$

We replace equality constraint with inequality constraint $C_l \geq C_{l_0}$
Constraint is enforced using penalty approach

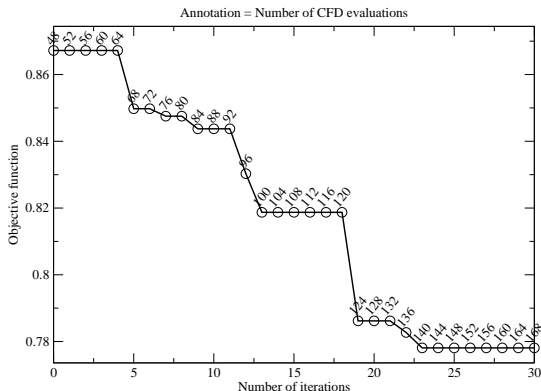
Unconstrained problem

$$\min \frac{C_d}{C_{d_0}} + 10^4 \max \left(0, 1 - \frac{C_l}{C_{l_0}} \right)$$

At convergence, we recover $C_l \approx C_{l_0}$

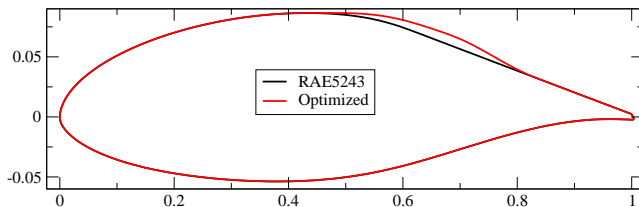
Optimization test

- 5 design variables
- Initial database of 48 using LHS
- 4 merit functions based on statistical lower bound with $\kappa = 0, 1, 2, 3$
- Gaussian process models
- Merit functions minimized using PSO



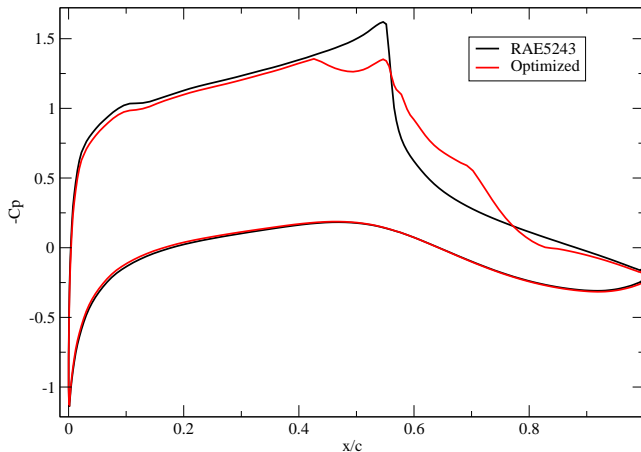
Shape parameters

Case	X_{cr}	X_{bl}	X_{br}	$\Delta Y_h \times 10^{-3}$
Present	0.688	0.399	0.257	8.578
Qin et al.	0.597	0.313	0.206	5.900

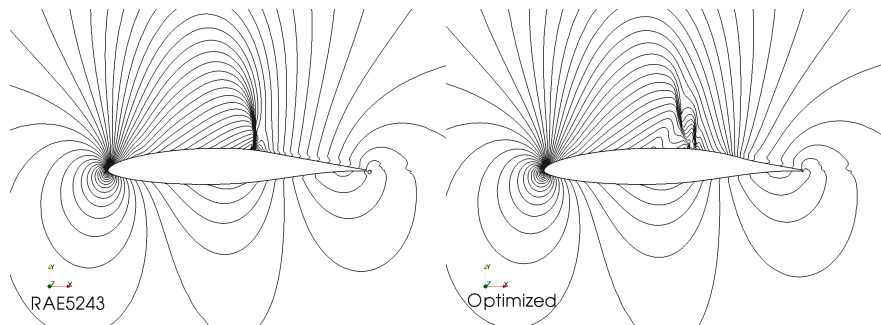


Force and Pressure coefficient

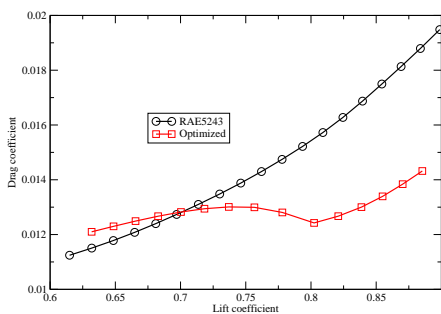
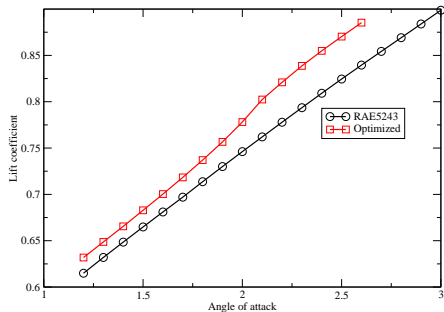
Case	C_d	ΔC_d	C_{d_p}	C_{d_v}	C_l	AOA
Present	0.01266	-22.2%	0.00680	0.00586	0.8204	2.19
Qin et al.	0.01326	-18.2%	0.00756	0.00570	0.82	-



Pressure contours

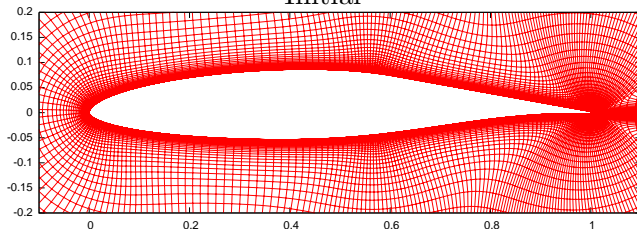


Lift curve and drag polar

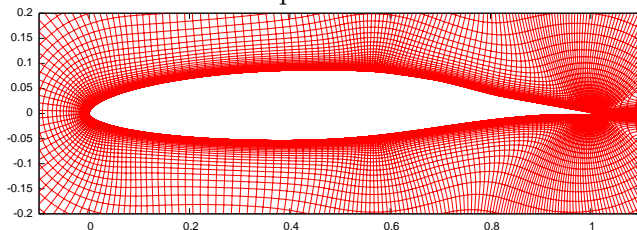


Close-up view of grids

Initial

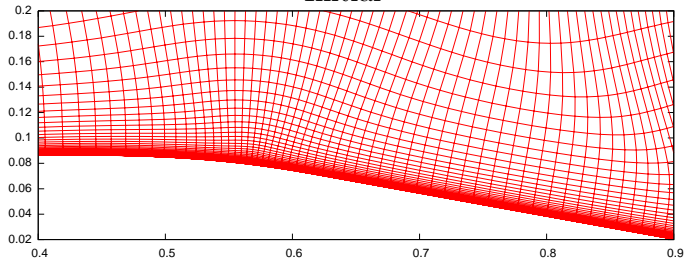


Optimized



Close-up view of grids

Initial



Optimized

